PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 62 APRIL, 1936 No. 4, PART 1

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except June and July, at 99-129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers, Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
A Direct Method of Moment Distribution. *T. Y. Lin*	Dec., 1934	
Discussion	Mar., May, Aug., 1935	Clo
Photo-Elastic Determination of Shrinkage Stresses. *Howard G. Smits*	May, 1935	
Discussion	Sept., Oct., 1935, Mar., 1936	Clo
The Shear-Area Method. *Horace B. Compton and Clayton O. Dohrenwend*	May, 1935	
Discussion (Authors' closure)	Aug., Oct., Dec., 1935, Apr., 1936	Clo
Flood-Stage Records of the River Nile. *C. S. Jarvis*	Aug., 1935	
Discussion	Nov., Dec., 1935, Jan., 1936	Apr., 19
Distribution of Stresses under a Foundation. *A. E. Cummings*	Aug., 1935	
Discussion	Oct., Nov., Dec., 1935, Jan., 1936	Apr., 19
Failure Theories of Materials Subjected to Combined Stresses. *Joseph Marin*	Aug., 1935	
Discussion (Author's closure)	Oct., Nov., Dec., 1935, Apr., 1936	Clo
Adaptation of Venturi Flumes to Flow Measurements in Conduits. *Harold K. Palmer and Fred D. Bowlus*	Sept., 1935	
Discussion	Nov., 1935, Feb., Apr., 1936	Apr., 19
The Stress Function and Photo-Elasticity Applied to Dams. *John H. A. Brahtz*	Sept., 1935	
Discussion	Nov., Dec., 1935, Jan., Feb., Apr., 1936	Apr., 19
Flood and Erosion Control Problems and Their Solution. *E. Courtland Eaton*	Sept., 1935	
Discussion	Nov., Dec., 1935, Feb., Mar., 1936	Apr., 19
The Relation of Analysis to Structural Design. *Hardy Cross*	Oct., 1935	
Discussion	Dec., 1935, Jan., Mar., Apr., 1936	May, 19
Tunnel and Penstock Tests at Chelan Station, Washington. *Ellery R. Fosdick*	Oct., 1935	
Discussion	Feb., Mar., 1936	May, 19
Tapered Structural Members: An Analytical Treatment. *Walter H. Weiskopf and John W. Pickworth*	Oct., 1935	
Discussion	Feb., Mar., 1936	May, 19
Proposed Improvements of the Cape Cod Canal. *E. C. Harwood*	Oct., 1935	
Discussion	Dec., 1935, Feb., 1936	May, 19
Influence of Diversion on the Mississippi and Atchafalaya Rivers. *E. F. Salisbury*	Nov., 1935	
Discussion	Apr., 1936	May, 19
Stable Channels in Erodible Material. *E. W. Lane*	Nov., 1935	
Discussion	Feb., Apr., 1936	May, 19
Truss Deflections: The Panel Deflection Method. *Louis H. Shoemaker*	Nov., 1935	
Discussion	Jan., Feb., Mar., Apr., 1936	May, 19
Lateral Pile-Loading Tests. *Lawrence B. Feagin*	Nov., 1935	
Discussion	Nov., 1935, Jan., Feb., 1936	May, 19
Sedimentation in Quiescent and Turbulent Basins. *J. J. Slade Jr*	Dec., 1935	
Discussion	Feb., 1936	May, 19
Wind Stresses in Reinforced Concrete Arch Bridges. *A. A. Eremin*	Dec., 1935	
Discussion	Apr., 1936	May, 19
Successive Eliminations of Unknowns in the Slope Deflection Method. *John B. Wilbur*	Dec., 1935	
Discussion	Mar., Apr., 1936	May, 19
Reinforced Concrete Members Under Direct Tension and Bending. *D. B. Gumensky*	Dec., 1935	
Discussion	Mar., Apr., 1936	May, 19
Progress Report of the Committee of the Irrigation Division on the Conservation of Water	Dec., 1935	
Discussion	Apr., 1936	Uncerta
Tall Building Frames Studied by Means of Mechanical Models. *Francis P. Witmer and Harry H. Bonner*	Jan., 1936	
Discussion	Apr., 1936	Aug., 19
Modern Conceptions of the Mechanics of Fluid Turbulence. *Hunter Rouse*	Jan., 1936	
Discussion	Apr., 1936	Aug., 19
Comparison of Sluice-Gate Discharge in Model and Prototype. *Fred William Blaisdell*	Jan., 1936	Aug., 19
Behavior of Stationary Wire Ropes in Tension and Bending. *Douglas M. Stewart*	Feb., 1936	Aug., 19
Varied Flow in Open Channels of Adverse Slope. *Arthur E. Matzke*	Feb., 1936	Aug., 19
Progress Report of Committee on Flood Protection Data	Feb., 1936	
Discussion	Apr., 1936	Uncerta
Progress Report of Committee of the City Planning Division on Equitable Zoning and Assessments for City Planning Projects	Feb., 1936	Uncerta
Progress Report of Committee of Engineering-Economics and Finance Division on Principles to Control Governmental Expenditures for Public Works	Feb., 1936	
Discussion	Apr., 1936	Uncerta
Surface and Sub-Surface Investigations, Quabbin Dams and Aqueduct; A Symposium	Mar., 1936	Aug., 19
Progress Report of the Committee of the Sanitary Engineering Division on Water Supply Engineering	Mar., 1936	Uncerta
Progress Report of Sub-Committee No. 2, Committee on Steel of the Structural Division on Structural Alloy and Heat-Treated Steels	Mar., 1936	Uncerta
Progress Report of Sub-Committee No. 31. Committee on Steel of the Structural Division on Wind-Bracing for Steel Buildings	Mar., 1936	Uncerta

Note.—The closing dates herein published. are final except when names of prospective dis-cussers are registered for special extension of time.

CONTENTS FOR APRIL, 1936

PAPERS

DISCUSSIONS

CONTENTS FOR APRIL, 1936 (*Continued*)

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

ADMINISTRATIVE CONTROL OF UNDERGROUND WATER: PHYSICAL AND LEGAL ASPECTS

By Harold Conkling,[1] M. Am. Soc. C. E.

Synopsis

Wells may be drilled in almost all valley areas of the United States with the expectation of securing sufficient water for household purposes and stock watering. Such supplies are of first importance in pioneer and farming communities, but are not of commercial importance. Neither legislative nor Court control of them has often been attempted. Usually it is only when supplies sufficient for irrigation, municipal, and industrial uses exist that engineering and law are involved. This paper deals only with the larger supplies.

A large percentage of the major cities of the United States is supplied with water produced from wells and probably most of the smaller ones are similarly supplied. Irrigation from wells exclusively increased 55.1% and, from wells as a supplemental supply, 177.5% in the 1919–1929 decade, whereas irrigation from surface streams decreased 6.3 per cent. This growing use of underground water has brought about much study of underground supplies throughout the United States and is causing changes in underground-water law. An important question that arises is whether the underground supplies should be placed under administrative control and the nature of legislative enactments creating such control.

This paper opens a discussion of the laws of underground waters, their consistency with those of surface waters, the possibility of enacting statutes for their administrative control, the form of such statutes, and the drawbacks and benefits of such control. The paper is not a treatise on underground-water law. This subject has been treated many times by minds much better qualified for the task than that of the writer. It is rather at attempt to reach a practical solution of a problem which, unlike most engineering problems, does not rest on physical laws alone, but on the laws of physics as interpreted

Note.—Discussion on this paper will be closed in August, 1936, *Proceedings*.

[1] Deputy State Engr., Sacramento, Calif.

by legal minds, and also on the fundamental law of property as applied to water. The foundation combines the legal with the material. Were this paper concerned with the design of a bridge the writer would not think of introducing into it the foundation concepts of elementary mechanics, but since it treats of the use of water, it is deemed necessary and advisable to outline the growth and development of water law so that the foundation for the conclusions will be apparent.

For those who may wish to go further into the subject, reference is made to Kinney on "Irrigation and Water Rights"; Wiel on "Water Rights in the Western States"; "Corpus Juris"; "American Law Reports"; and the great body of decisions of the Federal and State Courts.

Introduction

The usual physical situations in which ground-water of commercial importance is found, are as follows: (1) Definite underground channels; (2) percolating through basins; (3) underflow which may support a surface stream; (4) artesian areas or basins; and (5) alluvium or glacial till supplied by (*a*) irrigation; and (*b*) rainfall. These different physical situations are not clearly delimited and merge one into another. Underground supplies derived wholly from surface rainfall on valley floors are found in humid areas, but rarely, if at all, in arid areas. No essential difference exists between deep percolation from irrigation and that from rainfall. A description of typical instances would clarify the subject more than general descriptions.

Definite Underground Channels

If the underflow of a stream passing through bottom-lands several miles wide, bounded by bluffs, hills, or mountains, is regarded as a definite underground stream, it may be said that such streams, usable for irrigation, often occur in the West. The attitude of the Courts is not entirely clear as to the status of this supplementary flow, but in one case the Court of Arizona has defined it, definitely, as an underground stream. In most such streams in Arizona there is not perennial surface flow. The law covering such waters generally is the same as the law of surface waters, but nevertheless the subject is treated herein because such occurrences of underground water are important.

Percolation Through Basins

The San Gabriel Basin[2] lies immediately east of Los Angeles, Calif. In it are large areas devoted to citrus groves and walnut orchards and many towns and cities, including such famous municipalities as Long Beach and Pasadena. The basin consists of a mountain drainage area of 294 sq miles, an upper valley of 278 sq miles, called San Gabriel Valley, and a lower valley, known as the Coastal Plain, supplied by several streams of which the San Gabriel River

[2] "San Gabriel Investigations", by Harold Conkling, *Bulletin No.* 7, Div. of Water Rights, Dept. of Public Works, State of California.

is the largest. Of the entire area of San Gabriel Valley, 151 sq miles are using water for irrigation or for domestic and municipal purposes, and the Coastal Plain is likewise well developed. These are structural valleys filled with detritus from the mountains and surrounding hills.

A range of hills separates San Gabriel Valley and the Coastal Plain and through this barrier the San Gabriel River has cut a pass about two miles wide. Another range of hills almost faces the ocean and through this obstruction the stream has also cut two passes several miles wide. The alluvial plain below these hills faces directly on the ocean and is a true coastal plain as distinguished from the area above the lower range of hills which, although known as the Coastal Plain, is really a structural valley.

The summers are rainless and most of the precipitation occurs in the four months, December to March, inclusive. The annual rainfall in the San Gabriel Valley varies from about 26 in. near the mountains to 17 in. at the lower or southern side of the valley. In the Coastal Plain it is about 12 in. Rainfall is heavy at times and, although most of it is absorbed in the valleys, the steep impervious mountain slopes rising to elevations of about 7 000 ft and as high as 10 000 ft above sea level, shed the rain rapidly and cause large flashy floods. In such floods all the streams cross the pervious valley fill and flow into the ocean, but during most of the year the flows from the mountains are entirely absorbed if not used close to the mountains.

The stream beds in the valley are dry sandy washes during most of the year, but at the gap between the San Gabriel Valley and the Coastal Plain and, again, at the gap in the hills near the ocean, the underflow comes to the surface, creating a perennial stream of rising water, much larger at the upper gap than at the lower one.

The average slope of the surface of the San Gabriel Valley from the mountains to the outlet is about 50 ft to the mile and that of the Coastal Plain is about 12 ft to the mile. The water-table is 200 to 300 ft, and even 400 ft, below the surface in places near the mountains; it is at the surface through the outlet and is again 30 to 50 ft beneath the surface immediately below the gap. A few miles below the hills separating the San Gabriel Valley and the Coastal Plain artesian conditions begin and the piezometric level is above the surface between this point and the lower range of hills. Down stream from the latter point the water-table is again below the surface. The small summer flows from the mountains and the rising water are the only gravity supplies available for irrigation. All other supplies are from wells which, in general, have good yields.

The climate is subject to long cyclic fluctuations of rainfall. In 1934, the average water-table in the San Gabriel Valley was about 60 ft lower than in 1916, when the highest was recorded. In small basins partly cut off from the main area by underground formations the recessions have been much larger, whereas in the Coastal Plain and near the gap in the San Gabriel Valley they have been smaller. The area using water has increased from 32 000 acres in 1904 to 97 000 acres in 1934. The recession of the water-table is due partly to the increased draft to supply this demand, but mainly to lack of rainfall during the period, 1916 to 1934, and cumulative overdraft is not

indicated. Likewise, rising water at the outlet from the San Gabriel Valley has decreased from an estimated average of 164 cu ft per sec in 1916 to 38 cu ft per sec in 1934. Opportunity for percolation from the San Gabriel River is probably two to three times as much with the lowered water-table of 1934 as it was with the conditions of 1916, because of the greater distance the stream travels over an area beneath which the water-table is far below the surface. Opportunity for percolation of rainfall on the valley floor to the water-table is likewise greater than it was in 1916.

The particularly noteworthy points to bear in mind in a discussion of an underground-water law, is the decrease in rising water, the recession in the water-table, and the steadily increased use of water during the period when the water-table was dropping.

The characteristics of the San Gabriel Basin are typical of most of the coastal valleys of California south of San Francisco, and of the valleys or basins in which ground-water is found throughout the Great Basin which embraces parts of California, Oregon, Idaho, Nevada, Utah, and Wyoming. The major differences between Great Britain conditions and the San Gabriel Basin and others in California is the greater aridity in the Great Basin. As a consequence run-off is less, stream channels are often not cut through barriers down to the level now necessary to discharge water, and the water reaching the water-table is disposed of at the lower side of the basins by evaporation from lakes either ephemeral or constant, playas, and moist areas, instead of by the surface streams of rising water.

Underflow Which May Support a Surface Stream

The South Platte River has its source in South Park, Colo., on the eastern slope of the Continental Divide. After reaching the plain it flows northward through Denver, Colo., not far from the toe of the mountain and is joined by numerous tributaries, the last of importance being the Cache La Poudre. Then it flows eastward into Nebraska, is joined by the North Platte, and empties its waters into the Missouri River, near Omaha, Nebr.

Most of the area irrigated in Colorado from the South Platte is near the mountains in the high plains which begin their rise at the Missouri River and slope gradually upward to an elevation of 5 000 or 6 000 ft at the mountain toe where the precipitation is about 12 in. This rainfall increases, with distance eastward, to 30 in. at Omaha, occurring mainly in the growing season. Irrigation is practiced west of the 98th Meridian. The crops are practically the same throughout the irrigated area, consisting of grain, alfalfa, and a hoed crop which is corn in the eastern part, giving way to sugar-beets toward the west.

The South Platte River furnishes a supply for 1 233 000 irrigated acres in Colorado and 45 000 acres in Nebraska, a total of 1 268 000 acres. Of the area irrigated in Colorado about 8 000 acres are supplied from pumped wells. Of the 457 wells used for irrigation, 294 are in minor tributary valleys and only 4 in the valley of the main river. There are three wells in the valley in Nebraska.

Below the Cache La Poudre influx the river flows in a broad valley, generally ten miles or more wide, but sometimes narrowing to about two miles. The bottom-lands are only slightly above the water surface of the stream. The valley has been carved through the plain and the bottom refilled with porous detritus carried by the stream. The uplands are fertile, and their topography is suitable for irrigation, but as they rise from 150 to 350 ft above the river, it is not feasible either to pump for them or to reach them by gravity canals below the point where the river turns east. Wells drilled in the uplands do not reach sufficient porous water-bearing material to furnish more than domestic or stock supplies. The wells in the valley generally find porous aquifers from which irrigation supplies can be produced. Although reservoir development for impounding the floods of the South Platte System is exceptionally large, considerable water escapes into the more humid eastern area in flood times. More pumping in the river bottom would provide storage space into which the floods would percolate and greater development could be made. At the same time, supplies to existing gravity diversions below would be reduced and the people dependent on them would also be forced to pump to secure their quota of water.

The principal points to bear in mind in the foregoing discussion are the possibilities of greater development by pumping in the river valleys and the greater expense to secure a water supply, which would be incurred by all interests below the pumping plants.

Artesian Areas or Basins

The Roswell Basin[3] is situated in Southeastern Mexico and is a narrow strip about 65 miles along the Pecos River, a tributary of the Rio Grande from the north. The original area of artesian flow was 663 sq miles, but by the winter of 1926–27 it had shrunk to 425 sq miles because of heavy draft. Under a considerably larger area to the west ground-water is under pressure but development is not economically feasible. The principal intake area lies still to the west and water for it is supplied by run-off from the Sacramento Mountains which are parallel to the river, the crest being about 80 miles distant. The streams from the mountains percolate into their beds in crossing this area, and the water finds its way to the artesian aquifers. This is the principal supply but there is an additional supply from deep percolation of rainfall on the intake area.

The Roswell Basin is not a structural basin, but is more nearly an artesian slope with the strata dipping from west to east. The permeable artesian strata consist of honeycombed and cavernous limestone, but moderate quantities of water are produced from sand strata and sandstone. Water is found also in the surface alluvium or valley fill derived from surface drainage, local precipitation, irrigation return, and upward leakage from the artesian strata. The Pecos River acts as a drain to the valley and its waters are diverted for irrigation in, and at points below, the artesian area.

[3] "Geology and Ground Water Resources of the Roswell Artesian Basin, New Mexico", by Albert G. Fiedler and S. Spencer Nye, *Water Supply Paper 639*, U. S. Geological Survey.

The principal crops are cotton, alfalfa, orchard products, and corn. The annual precipitation is 12 to 14 in. in the agricultural area, and the climate may be classed as semi-arid. The annual average temperature is from 59° to 63°, depending on the altitude which is from 3 100 to 3 600 ft above sea level.

The draft on the artesian basin evidently has been greater than the recharge, as the continuous loss of head on the artesian wells and the shrinkage in area of artesian flow testify. The safe annual yield is thought to be somewhat greater than 165 000 acre-ft, but the annual draft is now (1936) about 200 000 acre-ft, of which more than 100 000 acre-ft are required for the irrigation of crops now supplied from artesian wells. Various non-beneficial natural disposals of water are in process which, together with leakage from wells, account for the remainder of the 200 000 acre-ft. The non-beneficial disposals are decreasing as the pressure lowers.

Originally, large springs flowed in the vicinity of Roswell, but these no longer exist as the water formerly issuing from them is now diverted underground to the artesian wells because of the depressed piezometric head. The area formerly irrigated by gravity by means of ditches is reduced to that which can be irrigated from "return from irrigation" and other minor sources. It is probable that other adjustments in the area irrigated by gravity from the river and tributaries will be made if the excess demands on the artesian basin continue.

The principal points for attention in the foregoing is the probable present overdraft, with consequent past and future decrease in surface flows, along a river formerly available for irrigation.

Alluvium Supplied by Deep Percolation from Irrigation

The Boise River has its source in the Sawtooth Range, in Idaho, and flows west into Snake River, a tributary of the Columbia River. At an early date, diversions were made on both sides of the river but principally on the south side. The normal flow in late summer is not sufficient to supply all water rights. In 1907, the United States Bureau of Reclamation started the Boise Project on the south side of the river paralleling older irrigated areas but at a higher elevation, and completed Arrowrock Reservoir about 1914. The rising water-table caused by deep percolation from irrigation on the Boise Project threatened ruin to some of the land on the Project and to a considerable area of the older irrigated lands along the lower river and on the south side of it. Surface drains to the river were installed and the land was thus protected. The discharge from the drains is used by rights diverting from the river below them during the irrigation season and water formerly allowed to flow down river from the head-waters is now diverted farther up stream by other rights. During the winter the drainage water wastes into the Snake River.

Some, but no great quantity, of underground water has been pumped for irrigation from this area of high water-table, but water shortages in recent years have forced the consideration of greater development of this source. If a sufficient quantity were pumped, the ground-water would be lowered

and the waste from drains in the winter would decrease, or perhaps cease entirely. The rights down stream might not have a full supply from drains in such a case and, if so, would have to be supplied with discharge direct from the head-waters.

The principal point to observe in this situation is that a greater area could have been developed or a better supply to the present area could have been given if, instead of building surface drains, wells and pumping plants had been installed to accomplish the same thing (assuming the existence of the necessary porous aquifers). The cost to the farmer of using this water would have been greater also, due largely to the subsidy on capital costs given in the financing of Federal irrigation projects, but the additional cost would have been spread over a large area. Under the appropriative doctrine if the proprietors of any land not now irrigated, or to be irrigated as a part of the Boise Project, should attempt to use this water, they might find the cost excessive because of the upset to the existing situation. As a supplemental supply to existing projects either over-lying the underground basin, or near-by, this water might prove very valuable, and might be secured without great legal difficulty.

Alluvium or Glacial Till Supplied by Rainfall.—In New York State, Long Island is marked by a range of hills from east to west forming the backbone of the Island and rising about 100 to 200 ft above sea level[4]. From these hills streams drain both north and south into the ocean. The surface soils are of glacial origin of particularly porous nature on the south side of the Island. The rainfall averages about 45 in. annually and, except when the ground is frozen, most of it is absorbed by the soil. Perhaps 40% or 50% reaches the water-table which, in most of the area, is below the root zone. The water-table slopes toward the sea and probably most of the underground water reaches the sea without coming to the surface. Near the stream channels, carved deeply by large ancient glacial streams, the water-table slopes toward the valleys and they act as drains. Other surface disposals are by evaporation from ponds and swamps where the water-table intercepts the surface.

As shown by the water development of the Borough of Brooklyn, N. Y., on the western end of Long Island, wells yield heavily. As irrigation is not practiced, except perhaps in a very limited way, there is little draft on the underground water except in the western end, to supply Brooklyn. If average annual deep percolation from rain averages about 20 in., as estimated, the yield from the approximate 500 sq miles of Southern Long Island would be very large and suited to the requirements of a large municipal center which requirement is the only one of magnitude for consumptive use in humid climates.

The Long Island situation is of interest because, aside from the western end used by the Borough of Brooklyn, the underground water resources are practically undeveloped, although they have been thoroughly explored as a supply to New York City. This is an unusual situation since in most cases knowledge comes only after development.

[4] "Long Island Sources", Rept. of the Board of Water Supply of the City of New York, 1912.

Exploitation of the supply would lower the water-table and thus create additional storage space underground for the wet years. Movement toward the ocean would decrease and salt water from the ocean would move farther under the land as pressure was removed. The discharge of streams fed from ground-water would decrease and ponds and swampy areas would diminish in size. If, instead of the ocean, a stream flowed along the southern shore its flow would be decreased.

The area is typical of those in humid regions suitable for large development except that some are tributary to a stream of magnitude instead of to the ocean.

Underground Water in General

Aside from the flood flows of a stream, its discharge is the result of precipitation on the tributary water-shed which has percolated below the root zone of vegetation and moved underground in the direction of the slope, to the stream (or other body of surface water). Underground flow is merely an evidence of water on its way to the surface. It may be used either directly from underground or after it has found a stream. It is the same water in either case, but in the one it has been brought to the surface by Nature, and in the other, by Man; but if Man had not interfered, Nature would have done the job at some lower elevation.

Any withdrawal of water from the aquifers causes a decrease in the water reaching the stream. It may be the floods that are decreased, inasmuch as the withdrawals will provide additional space into which the rainfall may percolate. On the other hand, the underflow may be decreased and thus the low water of the stream diminished. Water moves very slowly underground, its rate depending on slope and on porosity and thickness of the aquifers. If pumping withdrawal occurs at a sufficient distance from the surface outlet it may be that several rains, or even annual rainy seasons, may occur before the depressed water-table caused by pumping reaches the outlet; and, in such cases, sufficient replenishment may occur to restore the former underground supply completely. Such replenishment is unlikely in arid regions, but in humid regions the longer the period the more likely that the depletion will be made up by percolation which would not have occurred if the withdrawal had not been made. Likewise, the smaller the quantity of water pumped the more likely it is that it will be replenished. Pumping from the underflow of a surface stream would undoubtedly decrease the surface flow for a distance below the pumps, but might have no effect on it farther down stream.

Natural disposal of underground water must be in equilibrium with the recharge over a term of years, but the water-table constantly fluctuates with the wetness of the season, or cycle of seasons, because disposal is more uniform than recharge. For a time, perhaps many years after pumping draft begins, disposal continues as before, and the water-table in the general vicinity of the pumping lowers with comparative rapidity until natural disposal is decreased by the quantity of water pumped unless, as may occur (especially in humid regions), the draft is made good by additional recharge. If the new draft is not greater than natural disposal the water-table assumes equi-

librium at this new lower level. If it is greater, pumping draft must eventually decrease to equal recharge and again equilibrium will be established. In any event the stream below would be decreased.

In arid regions, especially, evaporation may be so great and the supply so small that disposal is accomplished from wet areas in the lower levels of the valley, and no surfaie stream issues from it. In such cases the lowered water-table caused by pumping merely decreases the evaporation.

In considering the different physical situations heretofore described in which ground-water of commercial importance is found, it is discovered that there are no essential differences. In some cases (such as that of Long Island and closed valleys in arid regions where the disposal is, in the one case mainly into the sea and, in the other, into an evaporating area) pumping would produce no general effect on a stream system but would nevertheless upset local conditions by lowering the water-table. In humid regions, where water-courses are more frequent than in arid regions, the distance between the recharge area and the point of disposal may be small and the effect on stream flow felt more immediately. On the other hand, greater probability of additional recharge below the pumped area exists in humid regions. Each situation is a separate study which must be exhaustive and widespread to reach reliable conclusions.

Ordinarily, the exploitation of an underground source has proceeded without supervision or knowledge on the part of those using the water. Such knowledge comes usually after a series of lawsuits to enjoin exploitation, or when alarm is caused by the receding water-table. A notable exception is the survey of the underground supply of Long Island previously mentioned. Presumably similar surveys have been made by other large organizations in seeking a water supply. The United States Geological Survey has done, and is doing, much work throughout the United States on investigations largely in co-operation with States or local entities, but because of paucity of funds its work is naturally most intensive in those areas where exploitation has caused alarm. Probably State supervisory boards are doing similar work in many States. In California, the State Division of Water Resources is doing considerable investigational work, but as the use of underground water in California far exceeds that in all the other States of the Union its work is likewise being done in regions already overdrawn, or where alarm has been caused by lowered water-table.

Extent of Underground Water Use

In this paper it is implicit that use of underground water for ordinary domestic or stock supplies is not considered. This draft is insignificant and has never, to any extent, been a matter of legislative or Court control. In humid regions (that is, in regions of the United States east of the 98th Meridian) underground water is exploited to the point where far-reaching effects are produced only to secure supplies for metropolitan areas and information as to the extent of such exploitation has not been found. West of the 98th Meridian the climate in the regions where agriculture is developed is

arid, semi-arid, or sub-humid, and irrigation is required. It is in part of this region that the possibilities, proportionately to those of the East, are highly exploited, and in some cases over-exploited.

There are more statistical data in available form for the West than for the remainder of the United States, and these data are summarized herein in tabular form. Table 1 contains data on irrigation for the years 1919 and 1929 for each of the nineteen States in which it is practiced.

This information is selected from a *Bulletin* of the U. S. Bureau of the Census, published in 1930[5]. In addition, the *Bulletin* reports an item under "Supplements from Pumped Streams, Pumped Wells, and Flowing Wells." The total shown for pumped wells[6] is 293 026 acres, of which 287 136 acres are in California and the remainder scattered through the other States. The same information is not given for 1919 and, therefore, it is not considered in Table 1. If it were added, the total for California, irrigated partly from wells, would be 1 070 318 acres in 1928, whereas the total for other States would be only slightly affected.

The total area irrigated is given by the Census as 19 191 761 acres in 1919 and 19 547 544 acres in 1929. The area in Table 1 covering three different classes of irrigation is 17 745 182 and 18 189 425 acres, or 92.5 and 93.0% of the total, respectively. The remainder not covered by Table 1 is irrigated from springs, lakes, etc., and from supplementary well supplies not owned by the owners of the surface rights. Table 1 gives: (1) The area receiving its entire supply from streams; (2) the area receiving its entire supply from wells; and (3) the area receiving a supply from streams supplemented by wells. The total shows for the decade a decrease of 6.3% in the first class and increases of 55.1% and 177.5% in the second and third classes. When California is excluded the increase in irrigation from wells becomes much less impressive. The decrease in the area irrigated from streams exclusively is then 2.6%; the increase in the area irrigated from wells exclusively is only 18.0%; and the areas with a supplemental supply from wells shows an increase of only 20.0 instead of 177.5 per cent.

Even this increase indicates a vitality in the growth of the use of underground water which does not exist in the case of irrigation from streams, or gravity irrigation, as it is commonly called. The use of underground water has gone forward in spite of conditions adverse to agricultural development which existed during the decade. The causes for this increase are probably: (1) Pressure for new development in certain areas in spite of over-development of agriculture in the United States and even in the State in which the new development is made; (2) failure of gravity supplies due to drought, or to additional use of the stream at higher elevations; (3) increased efficiency of pumps; and (4) lower power cost.

A clearer picture of the whole may be made by roughly grouping the arid States in accord with difference in economic, physical, and climatic conditions to determine in what general areas the increase has taken place and to lay the basis of an estimate for the future.

[5] "Irrigation of Agricultural Lands", U. S. Bureau of the Census, 1930, Tables 11 and 12.

[6] *Loc. cit.*, Table 37.

TABLE 1.—COMPARISON OF AREA (IN ACRES) IRRIGATED IN THE UNITED STATES

State	Areas Receiving Entire Supplies from: Streams		Areas Receiving Entire Supplies from: Wells		Percentage Change from 1919 to 1929		Areas Supplied from Streams with Supplemental Well Supply		Percentage change from 1919 to 1929
	1919	1929	1919	1929	Streams	Wells	1919	1929	
Arizona	196 453	170 797	41 810	106 002	—13.1	153.5	218 324	292 721	32.5
Arkansas	6 129	1 502	135 260	142 978	—75.5	5.7	250	....	—100.0
Colorado	3 050 964	3 169 559	14 390	15 929	3.9	10.7	84 138	17 656	—79.0
Idaho	2 384 010	2 029 016	1 545	5 569	—14.9	260.5	2 284	74 667	3 170.0
Kansas	32 137	56 412	13 285	11 651	75.5	—12.3	1 540	405	—73.7
Louisiana	271 152	259 001	155 575	175 787	—4.5	13.0	10 045	....	—100.0
Montana	1 550 827	1 487 751	351	1 064	—4.1	203.1	6 223	2 694	—56.6
Nebraska	437 532	503 653	546	23 452	15.1	....	115	70	—39.2
Nevada	470 179	395 236	1 171	3 426	—15.9	192.6	5 039	4 534	—10.0
New Mexico	434 368	436 955	52 295	58 115	0.6	11.1	2 026	1 015	—50.0
North Dakota	11 499	8 253	....	....	—28.2	.....	....	....	....
Oklahoma	2 710	675	125	63	—75.1	—49.6	....	....	....
Oregon	851 183	739 569	2 405	3 891	—13.1	61.8	305	3 322	1 010.0
South Dakota	93 360	65 916	130	528	—29.4	306.2	520	160	—69.2
Texas	495 870	699 146	44 466	62 624	41.0	40.8	499	850	90.5
Utah	1 116 130	1 040 577	12 394	19 655	— 6.8	58.6	662	3 520	431.0
Washington	471 145	450 067	20 665	20 995	—4.5	1.6	2 856	708	—75.3
Wyoming	1 157 121	1 183 252	166	320	2.3	92.8	400	137	—65.4
Sub-total *	13 032 769	12 697 337	496 576	586 775	—2.6	18.0	335 226	402 459	20.0
California	2 920 396	2 254 712	868 060	1 464 960	—22.8	68.8	92 152	783 182	641.0
Total	15 953 165	14 952 049	1 364 639	2 117 012	—6.3	55.1	427 378	1 185 641	177.5

* Exclusive of California.

Group 1 consists of Louisiana, Arkansas, and Texas (southeastern part). Although Western Texas is arid, the eastern part, with all of Arkansas and Louisiana, is humid. Irrigation by pumping from wells is practiced for rice even in the most humid portions of each of these States, as this crop should be partly submerged after a certain stage of its growth. Rice is grown on leveled land on which water may be ponded by dikes surrounding or going through the fields. In the section of this group of States where rice is grown, the terrain is so flat that gravity irrigation is impracticable in most cases and recourse must be had to wells or to pumping from streams. The ground-water supply is not sufficient to maintain the present draft in certain of the areas. The acreage in rice depends on the rice market. Future increase in the acreage irrigated from wells may not be large as it is unlikely that there will be any considerable failure of stream supplies making it necessary to substitute wells.

Group 2 consists of Montana, Wyoming, North Dakota, and South Dakota. In all four States only 1 912 acres were irrigated from wells exclusively in 1929. Although this is 196% increase from the 647 acres thus irrigated in 1919, the total is insignificant. The climate of these States is not conducive to crops which can stand the comparatively large annual expenditure required for pumping from wells, and it is believed improbable that the total cost will ever be so large as to cause it to be a problem.

Group 3 consists of Colorado, Kansas, and Nebraska. Colorado east of the Rocky Mountains is an area of high plains with an eastern gradient which continues through Kansas and Nebraska to the Missouri and Mississippi Rivers. The eastern limit of irrigation is at about Longitude 98° W, west of which lies about three-fourths of the length of Nebraska and two-thirds the length of Kansas. The principal streams from the eastern slope of the Rockies in Colorado flow on through Nebraska and Kansas. Precipitation occurs principally in the growing season. Both temperature and precipitation increase to the east and south. Crops raised by irrigation are quite similar over the entire area, but corn—the hoed crop in the eastern part of the area—gives way to sugar-beets as one goes west. Such irrigation from wells as occurs is practiced entirely in the broad bottom-lands of the streams. It is not feasible for the bordering plains. The cost of securing water in the bottom-lands is small, as the water-table is high and other conditions are favorable. Physical conditions are favorable to an increase in the area watered from wells. Surface supplies from the principal stream are unreliable especially as the distance from the mountains increases and until the humid section is reached, but there should be ample underground water for any demand that is likely to be made upon it.

Group 4 consists of Idaho, Utah, and Nevada. Except as to Northern Idaho these States are arid. The climate in a considerable area of each State is conducive to high crop values. Underground water is found: (1) In isolated basins from which the water that enters them is dissipated without surface outflow; (2) in larger basins, more prolifically supplied, from which issue streams originating in the ground-water; and (3) along river bottoms and under irrigation projects. Physical conditions are favorable to an

increase in the area supplied from wells, and in some basins of Utah the demand is said now to exceed replenishment. In 1919, the area thus supplied in this group was 15 000 acres and the increase by 1929 was 90 per cent.

Group 5 consists of New Mexico and Arizona. The southern part of these States is characterized by an intensely hot and arid climate. Irrigation from wells in New Mexico is largely in the great artesian areas. In Arizona, to a large extent it is along the Salt River and other tributaries of the Gila where irrigation has increased the ground-water supplies. Water is pumped, however, from the bottom-lands and valleys formed by the principal streams and from closed basins. The physical conditions are favorable to the extension of the area irrigated from wells.

Group 6 consists of Oregon and Washington. West of the Cascade Mountains irrigation is not practiced, but east of these mountains both States are arid. The climate decreases in aridity east from the Columbia River, in Washington. Some development of well irrigation exclusively is occurring in Eastern Oregon from basins, but in Washington little extension is being made.

Group 7 consists of California alone. Almost 75% of the total area in the United States irrigated from wells exclusively, and 66% of that which uses wells for a supplemental supply is found in California. To a limited extent the areas in which ground-water exists are found along the flood-plains of the streams, but most of the ground-water supply is found in the coastal valleys and in the great central valley. These valleys are filled with detritus and are supplied by percolation from streams crossing them and rainfall on the valley floor. Some areas are now over-developed. It is to be expected that only a small percentage of increase will take place in the future in the area of the

TABLE 2.—AREA IRRIGATED EXCLUSIVELY FROM WELLS, BY GROUPS OF STATES

Group No.	State	Remarks	Area, in Acres: 1919	Area, in Acres: 1929	Percentage increase
1	Louisiana......... Arkansas......... Texas...........	Humid climate; water used for rice growing.	335 301	381 389	13.7
2	Montana......... Wyoming......... North Dakota.... South Dakota.....	Cold climate; expensive pumping impossible.	647	1 912	196.0
3	Colorado......... Kansas........... Nebraska.........	Well suited to pumping from wells along streams. Much additional development possible.	28 221	51 032	80.9
4	Idaho............ Utah............. Nevada..........	Well suited to pumping in parts of this group. Considerable development possible.	15 110	28 650	89.6
5	New Mexico...... Arizona..........	Climate suitable to expensive development. Water naturally available is limited.	94 105	164 117	57.3
6	Oregon........... Washington.......	Climate favorable over great part, but ground-water conditions not suitable for large development.	23 070	24 886	8.0
7	California........	Ground-water over exploited in parts and must be replenished by importation if present development maintained.	828 060	1 464 960	68.8

State irrigated from wells, because most of the development possible has occurred unless (as is proposed for some areas) water is imported for artificial recharge.

Table 2 shows the comparison of area in each group irrigated from wells exclusively in 1919 and 1929. It is in Groups 3, 4, and 5, and possibly in Group 6, that climate and physical conditions point to further development with possibilities of over-development and attendant legal conflicts, such as have occurred already in some States, and particularly in California.

Water Law in General

Any legislative acts concerning underground water would presumably follow closely any existing legislation as to surface waters in the jurisdiction wherein the enactment is made, and it is desirable, therefore, to inquire into the law of surface waters in the various States before discussing that of underground water, in order to discover something of its nature. The law of underground waters, as now developed, is less than a century old, whereas the law of surface waters of the United States had its antecedents and development in the Roman Empire; hence for intelligent discussion reference must first be had to the law of surface waters. The civil law of the Romans is the foundation for the common law of England, which was adopted by each of the jurisdictions of the United States that came in after the original Thirteen Colonies where not incompatible with economic conditions prevailing in the new country. It appears probable, however, that the common law of England on water was taken from early American decisions which were derived from the Code Napoleon, which, in turn, was derived from the Roman law.

The doctrine of surface water originally made it the common property of the public, just as air and fish in the water—economically a "free good"; but because private ownership of land came to exist, it follows that the public is excluded from the use of the water except in the case of navigable streams and lakes, or the sea, and, consequently, the stream and the lake are the common property only of those who own lands bordering them. As a final outgrowth (in this country, at least) the common law doctrine of riparian rights permits each owner of riparian lands to use the natural flow of the stream on his riparian land in any reasonable way. In other words each riparian owner has a right correlative with, and equal to (proportionate to his holdings of riparian land), that of every other riparian owner. Riparian rights do not attach to all land in the water-shed, but only to the extent of the holding (which has never been severed) of lands bordering the stream, and even then not past the water-shed boundary. Priority of right does not exist; nor does the right exist to store water for use in a later season. Use must be made of the stream in its natural regimen if at all.

Water may be appropriated for use, however, on distant non-riparian lands in all jurisdictions. This is at the sufferance of the riparian owner where the riparian right is recognized and, if he insists, after compensation for any damage he may suffer if he makes his claim before the period of limitation

forecloses it. If no damage is suffered no claim can be made, as the riparian right, similar to the appropriative right, is the right to use, but not to hold, unused water against others who may have use for it. The definition of the use for riparian rights varies with the jurisdiction and is probably very broad in some States. Appropriations could be made under the civil law of Rome and can also be made under the legal systems of all countries the laws of which derive from the civil law. Undoubtedly, this was the case also under all legal systems foreign to the Roman system, which were developed under civilizations where irrigation was necessary. Mexico does not seem to have recognized the riparian doctrine and from reading its water codes it is concluded that its officials exercised a free hand in granting appropriations. Most of the arid States of the United States have rejected the riparian doctrine and have substituted for it an appropriative procedure entirely, based on Mexican procedure probably, but adhering strictly to the doctrine of priority of right obtained with an appropriation.

In the arid States water is more important than land, since it is the limiting natural resource in development. Under the appropriative doctrine it is conceived that water is public property, but that the State owns it in trust for all the people, and it is not the property of only those who have access to it because of riparian holdings. These States have ruled that the right to use water should be exclusive instead of correlative. A water right is a property right and the exclusive appropriative doctrine is somewhat more akin to the law of real property than the riparian doctrine of water law. In the appropriative doctrine, as it now prevails in the States, he who is first in time is first in right and can insist on his full right as long as reasonably needed even if it takes all the water in the stream except that necessary for domestic and stock use in most cases. In sequence of priority all users take their full right as long as the water in the stream will yield it without infringing on prior rights.

Wyoming, Nevada, Utah, Colorado, Arizona, and New Mexico, which are wholly arid States, and Idaho, which is partly arid, abrogated the riparian doctrine and adopted the appropriative doctrine either by constitutional provision or by statute at an early date in their history. Montana by a Court decision uttered in 1921 appears to have joined this same category. In the States immediately east of the Rocky Mountains (that is, North Dakota, South Dakota, Nebraska, Kansas, Oklahoma, and Texas) irrigation is necessary or desirable only west of about the 98th Meridian; that is, in only about three-fourths or less of their area, and is, therefore, not so important as in the aforementioned States. Both the riparian and the appropriative doctrines exist in these States, although incompatible in theory; and yet, because the riparian right in the arid parts has been limited to reasonable use consonant to the definition of reasonable use in connection with appropriative rights, or has been limited in some other way, the conflict is not important. In Oregon, riparian rights as defined herein have been virtually abrogated and, in Washington, they still exist but are no longer of importance. In California, the basic doctrine is strongly riparian but an appropriative procedure has been superimposed on it. Riparian rights have been limited by constitutional

amendment (1928) to that reasonably required for beneficial use comparable to beneficial use under appropriative rights. This was upheld and interpreted by the Court in 1935 (Peabody, *et al.* *v.* City of Vallejo, 2 Cal. 2d 351).

The appropriative doctrine lends itself to administrative control more fully than the riparian doctrine. In all the aforementioned States, where the most arid conditions prevail, an elaborate procedure as to surface waters has been set up for receiving filings for appropriation, granting permits and licenses, adjudicating water rights by stream systems, patrolling streams, regulating head-gates, and, in general, seeing that each water user gets the water to which he is entitled. In the less arid Western States the procedure is not so much elaborated.

One point in surface-water law, especially important to this discussion, needs to be considered: "Is the diverter from the stream protected in his means of diversion?" Under the common law doctrine the riparian owner has a right correlative with that of every other riparian owner to the use of surface water; he possesses nothing exclusive. If one suffers from water shortage all must suffer. Under the appropriative doctrine, the user of surface water has an exclusive right, which may be exercised when sufficient water is in the stream no matter what may happen to those having rights junior to him except that, under certain conditions, stock water and domestic supplies for the junior rights take precedence over every other kind of use by the senior rights.

Does this correlativity and this exclusiveness extend to the means of diversion? In other words, can the riparian diverter claim that an undue burden has been placed upon him if other up-stream riparians, after his works are completed, take from the stream sufficient to make his diversion works useless and to require him to incur additional expense if he wishes to continue to use the water? Furthermore, can the appropriator using surface waters claim estoppel on projects up stream which, while not depriving the stream of sufficient water for his right, cause him additional expense to divert?

It is patent that all degrees of deprivation might be incurred in such a case from the most minor to one in which the cost of changing diversion methods would be so expensive as to be complete confiscation of property. Actually, this latter would rarely be the case, and it would seem just that protection be given against such contingency. Most cases are not so serious but nevertheless any additional expense would be partial confiscation unless recompense were made. On the other hand the "hurdle" faced by the individual who proposes to divert at an up-stream point would indeed be high if he had to compensate all down-stream riparians whose diversion works would have to be reconstructed because of his act. Development would be seriously curtailed.

It would seem that a right to an exclusive method of diversion would be incompatible with the riparian doctrine. The riparian doctrine is such that any owner up stream or down stream can take a part of the supply of the original riparian diverter and deprive him of his use. As compared to this, the benefits of a certain method of diversion would be of comparatively small moment. As there are only a small number of riparian rights being exercised

in the United States in locations where water is important, it is probable that the matter has never come to the attention of the Courts. At least, "Corpus Juris" does not list any cases exactly in point.

When it comes to appropriative rights it is found that two cases were decided at a comparatively early date—Shodde *v.* Twin Falls Land and Water Company (161 F. 43) Idaho, and Natoma W. and M. Co. *v.* Hancock (101 Cal. 42) California—which held that the appropriative right did not include a further right to insist on a certain means of diversion. In the first case, the force of the current turned the plaintiff's water-wheel which, in revolving, lifted water in buckets attached to the wheel. The water in these buckets was dumped into a canal and reached the land of the plaintiff. The Court held that the plaintiff had appropriated water for irrigation, had made no appropriation for power, and denied injunction when a dam below caused back-water at this wheel. The Natoma case is exactly in point and declared flatly that the plaintiff must change his diversion system when up-stream takings interfered with his ability to obtain water by means of existing works. No other cases have been found.

Legislative enactments or "water codes" of the various States contain nothing on protection of means of diversion. It would seem from this fact, and from the foregoing cases, that short of absolute or a very great degree of confiscation, protection is not given to means of diversion. Protection of the water right seems to be regarded as important in the "water codes", but not protection of the other phases of the property right. Constitutional provisions regarding confiscation of property as interpreted by the Courts are the protection in such cases.

Comparison—Appropriative and Riparian Doctrines

Both the appropriative doctrine and the riparian doctrine have faults and both have advantages. The appropriative doctrine, particularly is an attempt to give the closest approximation possible of the law of real property; but land is fixed in position and stable in area, whereas running water is evanescent and its quantity varies from day to day, from season to season, from year to year, and from cycle to cycle. Manifestly, with such an unstable natural resource, no such security of possession can be given as is the case with land; but the appropriative doctrine attempts to approximate it as closely as the nature of flowing streams and vagaries of climate permit. In times of normal flow the difficulties are not such as to be serious, but in cycles of unusual drought the deficiencies of the doctrine become apparent and these defects result from this very attempt to give stability. A preferred class of first users exists and all junior appropriators can be deprived of even partial supplies (except domestic and stock necessities under certain limitations) to give a full supply to the preferred class. This result of the doctrine has been masked on many streams by the reservoirs constructed by the U. S. Reclamation Bureau which in many cases although originally built for the new projects of the Bureau, have been found large enough to supplement such of the prior rights as were deficient.

The riparian doctrine recognizes the variability of water supply and prorates it to the riparian owners; and thus, in times of deficiency, all riparian users suffer equally just as in humid areas all suffer during drought. Nevertheless, the riparian owners are a preferred class, as is the early appropriator. The base is somewhat broader, however, and the preference much more limited. Since it deals only with streams in their natural regimen, reservoir development is impossible under it and recourse must be had to the doctrine of appropriation. The riparian doctrine creates uncertainty as to water rights and encourages resistance to non-riparian diversions; but where limited to reasonable use (as rights under this doctrine now have been in all arid areas of America), the resistance is not serious.

As interpreted by the Courts neither doctrine has been entirely satisfactory for present complex conditions and there is a tendency toward modification. Compacts between States have been consummated, or are in process of consummation, which allocate certain shares of water in the stream system to the different States regardless of priority. The Colorado River Compact is a notable example. In California, the terms of the act creating the Water Project Authority which relates to the Central Valley Project, introduces another new phase in water law in that priority of an entire water-shed is given. In California, also, streams have been adjudicated by zones in which the relation of the water rights of one zone to those in another has not been observed. Such an adjudication is very similar to the Colorado River Compact.

In the water conflicts between States the Supreme Court of the United States is the only judicial body having power to assume jurisdiction no matter what doctrine of water rights prevails in them. The Court has assumed this power and has shown a tendency toward making an "equitable" allocation of the unused water where the appropriative doctrine is not in force in both or in all of the contesting States, leaving present uses untouched. It has also shown a tendency to continue to hold such situations under its control with the apparent expectation of revising its allocation as development proceeds.

It seems probable that whatever system now exists will continue to exist for the smaller stream systems, in all jurisdictions, but that construction of reservoirs and modification of interpretation will finally occur by which a more uniform and flexible distribution of the unstable water supply will be secured. For the larger stream systems, special laws (either legislative or judicial) framed to meet the situation, are probable. In other words, the two doctrines will continue to meet the requirements of minor matters, but modifications of both are being made for the large developments.

Underground Water Law in General

Only in comparatively recent times has underground water become important. The first case used as a precedent by the Courts of the United States was decided in England in 1843 and the controlling case in 1857. In these decisions, underground water was considered as a mineral which could be

extracted by the owner of land at his will without recourse on the part of others who might be damaged by such extraction. All American Courts followed this rule for a time, but in 1862 the New Hampshire Court departed from it and promulgated the rule that the use of underground water must not be greater than reasonably necessary for the tract of land in which the water is produced. Under the former rule exportation of water could be made even if it damaged others dependent on the underground water either directly or indirectly, but in the newer rule such exportation was not allowed if damage resulted. This is termed the "doctrine of reasonable use", or the "American rule"; it is applied in a considerable number of the States in the humid sections of the United States whereas in the others the former doctrine of what may be termed "unreasonable use", or the English rule, still prevails. All Western jurisdictions, also, followed the English rule at first, but when greater knowledge of physical conditions came about through the use of underground water for irrigation, especially in California, the California Court (Katz *v.* Walkinshaw, 74 Pac. 766) introduced another doctrine which abrogated the English rule and expressly adopted a further and entirely logical development of the doctrine of reasonable use. This rule holds that each land owner overlying a basin has a right to the underground water co-equal and correlative with that of all other land owners overlying the same basin. In contradistinction to the doctrine of reasonable use this means that one land owner can not extract more than his proportion of the underground water even for use on land overlying the source when by so doing he encroaches on the right of another owner to do the same; nor can water be exported if injury to the overlying land owners results.

This places underground waters in California under the same doctrine as surface waters on riparian lands; that is, under the doctrine of correlative right. By later decisions, all the waters of a stream system, whether percolating and diffused underground; flowing underground in contact with a surface stream; flowing underground in a definite channel; or flowing as a surface stream or ponded in a lake, were treated in the same way, and the use of underground water was made correlative with the use of the surface stream.

The rule of underground water as promulgated in Katz *v.* Walkinshaw was adopted by the Courts in some Western jurisdictions in spite of the fundamental appropriative doctrine which exists in many of them. This rule is compatible with the riparian doctrine of surface streams, but is not compatible with the doctrine of exclusive appropriation of surface waters prevailing in the strictly arid States. Some other Western States still adhere to the English rule; thus, in many Western States, there are Court rulings which define the law for underground water either on the reasonable or the unreasonable use basis, and an entirely different legislative doctrine for the surface stream which may be fed by the underground basin, a condition that is certain to cause legal confusion where economic conditions permit the development of underground water.

A digest of Court decisions made in 1928 (55 A. L. R. 1390) lists thirty-one States (including the District of Columbia), mostly in the humid region, but including five in the arid region, in which former decisions are favorable

to the common law rule of absolute ownership of underground waters. This doctrine, as heretofore stated, was first challenged in 1862 in New Hampshire, and the doctrine of reasonable use was introduced. Not until 1900 did any other State concur, but in that year the Court of New York adopted the same rule and, in 1909, the Court of New Jersey followed. Of the States listed in the foregoing digest a total of six had reversed themselves and eight have not recorded decisions since 1900. Twelve States not listed in the digest have adopted the doctrine of reasonable use (Neb. L. B. 12; 191–6N, 1933). It appears that the trend in humid States is toward the American rule, but that many jurisdictions still hold to the English rule.

The status of the law as to underground waters in the various Western States (there are statutes in many States prohibiting the waste of artesian water, but this phase is not considered in this paper), is as follows:

Arizona.—There are no statutes on underground water in Arizona. The Court has ruled that such water is not subject to appropriation (Maricopa County Municipal Water District *v.* Southwest Cotton Co., 1931, 4 P. 2d 369), and, at first, reserved decision as to whether the English rule or the correlative doctrine which originated in California should prevail; but in 1934 (Fouryan *v.* Curtis, 29 P. 2d. 722), it decided in favor of the American doctrine of reasonable use—that is, that the water belongs to the land under which it is found and could be taken to distant lands if injury did not result to another on the stream system. In the case of Pima Farms *v.* Proctor (245 P. 369, 1926), the underground water in a stream valley several miles wide was in question. No surface flow exists except in flood times. The Court ruled that this was a definite underground channel (the litigants stipulated at the outset that the water involved was the immediate underflow of the river), that the prior user of underground water had a vested right to the maintenance of the water level, and that subsequent users must deliver water to him at no greater cost than had been incurred prior to the new use. This is similar to declaring that a prior appropriator of surface waters had a vested right in the means of diversion; in fact, the Court states that this is the law, but cites no decisions to that effect.

California.—There are no statutes on underground water in this State, but it may be appropriated by taking on sufferance of the overlying land owners. These appropriations ripen into a right after five years of open taking and are so recognized by the Courts.

No exclusive rights are permitted in underground waters except the appropriations previously noted. Cities and other municipal organizations are regarded as appropriators even if they are located directly above the underground basin. All overlying lands, and lands riparian to a stream where the percolating waters feed a surface stream, have correlative and equal rights to the stream system whether water is on the surface or underground (that is, percolating). A diversion from a surface stream made prior to 1914 may be an appropriation even if on riparian land, and, as such, prescription obtains against an underground water user below, who may be supplied wholly or in part by percolation from the surface stream.

The law of waters in California, both surface and underground, is highly developed and rests on reasonable use and the correlative doctrine of equal rights.

Colorado.—Underground water is not mentioned in Colorado statutes as subject to appropriation. The Courts have held that it is subject to appropriation, however, and subject to the same regulation as surface water. Rights under appropriation are in order of priority of filing on the stream in question. The Courts seem to be tending to the rule that in Colorado all waters, whether surface or underground, are presumably tributary to a surface channel, and that their taking is thus subject to prior appropriation of the surface stream.

In Colorado the taking by an underground user is stopped if it decreases the surface flow available to the down-stream user even if the down-stream user could sink pumps to the underflow and get a full supply. This amounts to a guaranty of the method of diversion previously discussed.

The theory of underground water law in Colorado is consistent with that of surface streams, but the application varies in the aforementioned particular. The waters of an entire stream system are treated as one.

Idaho.—Underground water is not specifically mentioned in statutes in Idaho. The Court concurs in the doctrine of appropriation of underground waters and states that it may be by the procedure of the water code, or by taking (Silkey *v.* Trego, 5 P. 2d, 1049, 1931). The subsequent appropriator of underground water must not lower the water-table from which the prior appropriator pumped (Noli *v.* Stonen, 26 P. 1112, 1933). In other words, the prior appropriator is protected in his means of diversion when underground water is in question.

Kansas.—All underground waters in the northwest quarter of the State are, by statute, subject to appropriation. Disputes on underground water have not been before the Court to any great extent, but so far decisions appear to be based on the English rule.

Montana.—There are no statutory provisions for the appropriation of underground water in Montana. Conflicts involving underground water have not been frequent. A case decided in 1912 followed the English rule (Ryan *v.* Quinlan, 124 P. 512).

Nebraska.—There is no legislation concerning underground water in this State, but in 1933 the Court declared in favor of the doctrine of reasonable use (Olson *v.* City of Wahoo, 124 Neb. 802).

Nevada.—The statutes provide that all water within the State, whether above or below the ground surface, belongs to the public and may be appropriated for beneficial use as provided in the act and in no other way; but it specifically eliminates percolating water, the course and boundaries of which are incapable of determination. Use of underground water is not great and details of administration have not been established by State authorities. There have been no recent Court decisions on the matter.

New Mexico.—In 1927, a statute as to appropriation of underground waters was passed in New Mexico, but was declared unconstitutional in 1929 because

of faulty title. In 1931, a new act, designed to satisfy the Court's objections to the first, was passed by the Legislature.

The statute applies to waters of underground streams, channels, artesian basins, reservoirs, or lakes having reasonably ascertainable boundaries; and declares them to be public waters and subject to appropriation. As a result of the definition, waters diffused and percolating toward a stream in the manner customary in humid countries, may not be included.

As a result of investigation the State Engineer has declared three basins as coming within the scope of the law. Two of these are basins in which the water is not under pressure, but in a condition such as that defined previously under the heading, "Percolation Through Basins." The other is the famous Roswell artesian basin previously mentioned.

The degree of co-ordination between procedure in the case of underground water and surface water in the same stream system is not apparent in the statute or the procedure outlined by rulings of the State Engineer. Apparently, the State Engineer proposes to accept filings only in those basins which have been examined and which have been declared to come within the purview of the statute. The Court has declared that adjudications should embrace both ground and surface water in one proceeding (El Paso and R. I. Ry. *v.* District Ct. 8 P. 2d. 1064, 1932).

North Dakota.—The statutes of North Dakota declare underground waters to be in the same ownership as the land on which they are found. No Court decisions have been made.

Oklahoma.—The statutes of this State make no mention of underground water and Court decisions on the question have not been found.

Oregon.—All waters, according to the Oregon statute of 1909, may be appropriated for beneficial use. In 1927, underground water east of the Cascades was declared subject to appropriation when it occurred in basins the boundaries of which could be defined with reasonable certainty. As finally amended in 1932 the statute as to underground waters still limits appropriation to the areas east of the Cascades and conforms to the law of New Mexico. Applicants for appropriations follow the general procedure outlined for surface waters. Development is proceeding east of the Cascades. There are no recent Court decisions as to the status of the underground water in Oregon.

South Dakota.—The statutes in force in South Dakota are "silent" on underground water. All Court decisions uphold the common law doctrine of absolute ownership and unreasonable use.

Texas.—There are no statutory enactments as to underground water in Texas. All Court decisions uphold the doctrine of absolute ownership of underground water.

Utah.—By a number of decisions, the Utah Court has held that owners of overlying land have co-equal and correlative rights in underground water (Katz *v.* Walkinshaw, *supra*) and also that such owner may export his *pro rata* share to distant points. The controlling case is Glover *v.* Utah Oil Refining Co. (218 Pac. 955, 1923). In the case of Wrathall *v.* Johnson (1935), the Court —in a peculiarly divided opinion—held that the use of underground water is

by appropriation even if on overlying land and is subject to the same restrictions as prevail for appropriations of surface water. This was a decision on demurrer and cannot be regarded as conclusive.

The Legislature of 1935 enacted a statute placing underground water in the same status as surface water; that is, as a right secured only by application for appropriation to the office of the State Engineer. This act follows a model recommended by the Association of Western State Engineers and is similar to the New Mexico statutes.

Washington.—All water in the State of Washington is declared to be subject to appropriation which would include underground water, but State officials consider that no authority is conferred on the State over such waters. One Court decision on underground water adopts the doctrine of reasonable use on overlying lands; another that underground water may not be taken to the injury of surface water diverters from the stream. The doctrine of the Courts is not well defined.

Wyoming.—No mention is made of underground water in the statutes of the State; nor are there recent Court decisions to clarify the law.

All Other States.—The only attempt at statutory control coming to the notice of the writer is in New York State where wells on Long Island drawing more than 100 000 gal per day are placed under the jurisdiction of the State Conservation Commission.

Summary.—Summing up the results of the foregoing examination, there are found to be four doctrines of law in the United States on which use of underground water is based: (1) Absolute ownership of water because of the ownership of the land beneath which the water is found, with no obligation to respect the rights of others, is herein termed the "doctrine of unreasonable use" or the English rule; (2) absolute ownership of water to the extent of reasonable use on the land beneath which the water is found, but with no right to export to distant land if by so doing damage is caused to another, is herein termed the "doctrine of reasonable use", or the American rule; (3) ownership, co-equal and correlative with that of every other land owner, of water lying over the basin, or riparian to a stream fed by water rising from the basin, is herein termed the California doctrine; and (4) entire lack of ownership on the part of the proprietor of the land, but ownership by the State instead—which allows use by appropriation under a procedure set by the State, or otherwise, and which is subject to prior rights of other users whether from the surface stream or from underground sources tributary to the stream—that is, the doctrine of prior appropriation. These differences are successive and cumulative impositions of control or, broadly speaking, the police power as found desirable because of the growing use of underground water, and as found possible because of increased knowledge of ground-water hydrology.

Unless analyzed these diverse doctrines would seem to entail endless confusion, but when it is remembered that there is also great diversity of climate in the United States and also great variation in the present stage of development and possibilities of future development, the probable confusion appears

not to be very great as a whole although potentially bad enough in limited areas. The worst legal confusion could result from lack of consistency between surface-water law and underground water law in the same general region; but, even where these laws are inconsistent, the climate may be such that costly development is impossible and, if so, the conflict may be more apparent than real.

Administration

Surface Streams.—From a stream of any extent, in arid regions, there are numerous diversions made for irrigation and it is obvious, in view of fluctuations of stream flow, that if use is large as compared to flow, chaos would result if these diversions were not controlled. In times when the flow is not sufficiently greater than the rights so that control is not needed, the administrative official allows to each diversion its right as determined by the law, Court decisions interpreting the law, Court decrees dealing with water rights in the stream, custom, and his own interpretation of the law. The decrees or the law not only determines the priority, but the amount of diversion. This administration is possible under both riparian and appropriative systems of law. Under the appropriative doctrine the administrator goes further and permits or denies new rights (if the law gives authority to deny), depending on his determination of the existence or non-existence of unappropriated water. In many States authority to deny is not given but control rests on the water master. So far as known no administrator denies a permit or in his policing of the stream closes a diversion because of difficulty which may be, or is, caused to a lower diverter as long as the water required by the right of the lower diverter will reach his point of diversion on the surface; but accompanying almost every surface stream is underflow which is legally a part of the stream system, and where this exists, it can be diverted by pumping so that even if the surface stream is all diverted at a point up stream, the lower user is not necessarily deprived of water. So far as known, however, administrators take no cognizance of this in their administration. This custom is based on Court decisions.

The Courts in all jurisdictions, in so far as can be found, have enjoined surface diversions which would have made it necessary for a prior and lower diverter of surface water to pump to secure water. It seems probable that this is the outgrowth of custom antedating the use of underground water, or it may be that a change from surface diversion to pumping diversion is unconsciously regarded as a rough dividing line between complete confiscation and endurable confiscation, discussed previously, in connection with surface water; or, it may be that, because the appropriation of surface water was made, it is regarded as necessary that surface water be preserved to the right. It is only recently that connection between surface water and underground water has been recognized legally. Formerly, the two were treated in separate and distinct doctrines of law, and still are, to a great extent. This matter has an important bearing on the development of underground water because it is probable that once control of the use of underground water is attempted,

both classes of water will be brought under the system which prevails as to surface water in those situations where both are involved but most statutes so far enacted for control seem to ignore this probability.

Underground Water.—The discussion of State laws under the heading, "Underground Water Law in General", indicates that there has been little legislation on underground water and that administrative control has been attempted in few jurisdictions. Under the headings, "Percolation Through Basins", "Underflow Which May Support a Surface Stream", "Artesian Areas or Basins", "Alluvium Supplied by Deep Percolation from Irrigation", and "Alluvium or Glacial Till Supplied by Rainfall", several typical situations were described in which underground water is of commercial importance. Consider the situation in San Gabriel Valley. No administrative control of the use of underground water is exercised in California, and all underground water rights are correlative one with another and with rights in the surface stream of the system. Consider the present condition had the surface rights at the lower end of the valley been empowered to enjoin the use of underground water at up-stream points in order that the surface flow might be maintained, as such rights probably are empowered in all appropriative doctrine States, or will be if the two systems of law are welded. In 1912, 32 000 acres were irrigated in the valley at up-stream points, whereas, in 1934, there were 97 000 acres, but there is no indication of long-time deficiency in supply in spite of the increase. This increase could not have occurred had such power existed. Had this additional draft not taken place the water-table would now be higher than it is; the capacity of the underground basin above the water-table would have been smaller than it is at present; and the amount of percolation from floods would have been much less. In short, unless these paramount rights had been extinguished, development would now be much less than it is and the water now percolating in flood time into the underground reservoir would be wasting into the ocean. On the other hand, those having surface water rights would secure it at less cost than they now do, as they have been forced to resort to pumping to secure part of their supply. The result of the existing law has been a more complete development and the greater unit cost which is generally incurred by recent developers has been spread more or less evenly over all users.

Consider the Roswell Basin: The water which supplies the artesian area of the Basin, in part at least, flowed through, or out of, the region as sustained flow usable for irrigation; but surface users have now been deprived of this water. In part, there may be recharge now from floods which was formerly wasted. If the paramount consideration had been to preserve the surface flow to satisfy rights therein, the Basin, in all probability, could not have been developed as it now is, and the irrigated area served by the stream would be less unless surface reservoir capacity had been substituted. If this is true, development of the Basin has gone forward because of lack of administrative control.

Consider the South Platte: The control of underground water has existed through Court decree, and encroachment upon surface rights has been inhibited. Some, but no great percentage of, stream discharge is now wasted

and most of this water could be captured were pumping allowed in the river bottom and were more acreage developed. Additional cost would be incurred by all users down stream from the location of pumping and the cost of new development would have been spread to some of the prior users; but to conserve this water by surface reservoirs would also cost money, which would be furnished by the direct beneficiaries, and perhaps construction of a reservoir would not be financially feasible for them.

The foregoing examples suffice to illustrate the results of administrative control under the concepts now existing. Clearly, development now exists which would not exist had control been in effect, yet, in many cases, in spite of this greater development, draft does not exceed supply. Whether this added development, with its attendant increased costs to early water users, is more desirable than a smaller development and lower costs is a question that need not be discussed in this paper, but where draft does exceed supply there can be little argument that the condition which exists is desirable.

Although control of surface waters is not simple, the administrator charged with the control of underground water finds himself involved in a vastly more complicated situation than he who administers only surface streams. He may be faced with the necessity of re-examining his concepts of what constitutes a water right. His decision must enter the field of economics from which he has been free in the administration of surface streams. Practically, no extensive use of underground water can be made in the West without diminishing the normal summer flow of surface streams below it where such flow exists and as these streams in most cases are appropriated the precedent of decisions will incline the Courts to protect the user of surface rights against depletion by underground-water users. If this is the case, administrative control merely means complete inhibition of pumping where surface streams are involved unless pumping occurs so far from the stream that the depressed water-table has no effect on the summer flow.

The Court of Idaho has already indicated by decisions, previously cited, that it considers that the first underground-water user has a right to compensation against later pumpers in the same basin, whose draft would lower the water-table. The Court of Arizona (Pima Farms *v.* Proctor, 245 P. 369) has adopted the same attitude as to water in a definite underground stream. Pumping cannot be done without lowering the water-table, and each additional draft causes additional lowering. Consequently, even without reference to surface rights, these rulings constitute absolute prohibition of the development of underground water in these jurisdictions, except in similar situations. Statutes embodying control by the water authority merely make it easier to accomplish the same result if the administrator either voluntarily adopts the same view, or is forced to do so by the Courts.

Administration of a surface stream involves only matter which is in plain sight on which, as compared to underground water, only a small amount of study is required. The problem is solved largely in the field and a set rule is followed if only priorities are considered. Administration of under-

ground water involves gathering and studying data. The work is in the office, and decision is not so easy. The cost of obtaining sufficient information to provide a basis for intelligent decision is considerable and this cost would not be justified where economic conditions do not permit extensive use of underground water and, from the same consideration, administrative control would not be justified.

If funds are sufficient for study, administrative control should lead to greater knowledge of hydrology and the education of engineers and Courts so that water law can be modified logically. If such changes are to be constructive the powers of the administrator must not be too hampered, and his function must at least be quasi-judicial. The increased responsibility requires greater power than ordinarily lies in the water administrator's office. Above all, it requires power to deny applications.

Summary, Discussion, and Conclusions

1.—The use of underground water in the United States is increasing continually. In humid regions this use is principally for municipalities, but in arid regions the draft is for both municipalities and for irrigation. In the East it is only in the vicinity of very large or very numerous municipalities that the draft is sufficient to be of concern and in the West it is only where physical and climatic conditions are favorable. There has been very little use of underground water in the Northwestern States; and there is no conceivable prospect for increase except in the arid or semi-arid portion of those Northern States favored by the Pacific slope climate.

2.—An underground basin contains a body of water that is moving; but, at the same time, it is a reservoir because it is moving very slowly and it may take several years for a unit of water to pass through it. Withdrawal of water from the underground reservoir decreases the quantity that must be disposed of by Nature unless conditions are such that the quantity pumped is replenished by percolation from stream beds, or by deep percolation from rainfall, neither of which would have occurred had not underground space been created by pumping. This replenishment, if it occurs, does so as Nature sees fit and is not under the control of Man. If the underground water is tributary to a stream, the stream flow will be decreased by a draft on the underground basin. This decrease may be either in the period of low water or of high water, or both, and its time is non-controllable.

3.—In humid regions there are few statutory enactments as to rights to use water. The body of Court decisions rules. Both the English and American rules exist for underground water, but the trend is toward the American rule in which ownership of the water is not vested absolutely with ownership of the surface. This is not on the same basis as the riparian doctrine in surface stream flow, but satisfies the requirement in most cases, as does also the English rule, apparently, where it prevails.

4.—All the arid States of the Union have instituted more or less far-reaching administrative control of surface waters in order to obviate the chaotic conditions which generally result without such control. Such legis-

lation was adopted by many very soon after Statehood was granted and, at that early period, the use of underground water was nil, or practically so. Probably, as a consequence of this condition, reference to underground water in the statutes of many of the States is lacking, inadequate, or must be inferred by the all-embracing wording of the statutes. In California, because of the clear-cut doctrine developed by the Court prior to the enactment of statutes regarding surface waters vesting the rights to underground water in overlying lands, reference to it was omitted from these statutes.

5.—In California, which is the only jurisdiction in the United States recognizing the riparian doctrine where the law of underground water has been fully harmonized with surface water law, each land owner having access to the usable water of the stream system, whether it is on the surface, or underground, has a right to a reasonable use of the water. If the owner whose land overlies an underground basin, in exerting his right, infringes upon a surface-water user's supply the latter has no recourse if the infringement is not excessive and he may be deprived of his surface right entirely if he can secure water by pumping. He is thus deprived only as the result of the exercise of another right equally as good, vested in all who have access.

On the other hand, in Colorado, which recognizes the appropriative doctrine instead of the riparian doctrine, and in which State the law of underground water appears to have been more fully correlated with the law of surface waters than in any other State except California, the owner whose land overlies a body of underground water is prohibited from using that water if such use would decrease the surface flow necessary to supply the appropriative rights of surface users.

The result of the legal doctrine recognized in California has been so far to give a free hand in the development of underground water. The result of the legal doctrine recognized in Colorado is practically complete prohibition of such development where rights to diversion from surface streams are involved, because the surface streams were used in most cases prior to the time that the use of underground water began.

6.—In the absence of definite statutory enactments the Courts have adopted diverse views as to the doctrine of law applying to underground water and in many jurisdictions even in the West the doctrine as enunciated by the Courts is not compatible with the doctrine of law prevailing for surface waters. There is a long line of decisions in the United States which vests with the owner of the surface the right to use water under his property although the right is not absolute but rather relative in most jurisdictions and the tendency is toward the relative doctrine. This right to use underground water seems a natural one and the Courts tend to promulgate it, but in a State where the theory of the appropriative doctrine prevails (State ownership of water), underground water should be regarded as belonging to the State if the law of underground water is to be compatible with that of surface water.

7.—It is notable that, in California and Colorado, the two States in which use of water for irrigation is largest (see Table 1), underground water and surface water are in the same system of law. In other words, the fact

that they are part of the same stream system is recognized. It seems entirely probable that this same development of law will come about in other States whether or not statutes enacted for the control of underground water ignore the interdependence. It would be convenient in many cases if this did not occur, but to continue to ignore, legally, the interconnection while at the same time putting administrative control of underground water into effect would seem to be an impossibility. The following Conclusions assume that the two classes of water will come under the same system of law.

8.—The appropriative doctrine (Conclusion 6) rejects the right of each individual to pump commercial quantities of water from his own land, and makes it necessary to secure a permit from the State to do so. In any effective administration the permit would be granted only after a survey of the entire situation and, since the appropriative doctrine seeks to preserve to prior users the full measure of their water right, would lead to rejection if the investigation disclosed that use of underground water would cause decrease in the water to which they are adjudged to have a right provided the concept prevails, that a water right on a surface stream cannot be satisfied by pumping from underflow.

9.—No doubt legislation on underground water in States recognizing the appropriative doctrine would be a powerful influence in future Court decisions, but if the legislation should be contrary to the doctrine of underground-water rights laid down by the Courts in sufficient previous decisions to make it well grounded, it might be declared unconstitutional in certain jurisdictions inasmuch as the existence of previous decisions means that development exists under the doctrine of law adopted by the Court.

10.—It is improbable, but not impossible, that legislation on underground water—even in some States recognizing the appropriative doctrine—might recognize that a right to use such water exists in the ownership of the overlying land, in which case prior use in surface streams fed by underground water would not constitute a right which could be exercised against the users of such water. Much more probable legislation, however, would place all water, whether surface or underground, on the same basis and under the control of the water authority.

11.—The water authorities of the arid States generally grant a permit to an applicant for the use of surface waters. This can be done because the streams are patrolled. If the project is constructed the diversion works can be closed when the surface flow is not sufficient for all prior rights; but when an underground supply feeds a surface stream it will be impractical to follow the same course on applications for rights to pump from the underground basin because the evidence of the effect of the draft on underground water is so long delayed, and because the time at which the effect will occur is uncontrollable. Action by the authorities (if it is intelligent) must have behind it the knowledge gained by a thorough investigation and if, in those States where the law of underground water is, or becomes, compatible with the surface-water law, this leads to the conclusion that the use of underground water will reduce the supply available to surface-water users, a permit would be denied if the surface right is paramount. There is no half-way point. At

present, practically all surface streams are fully used in the late summer and, on many of them, reservoirs have been constructed which impound all the flood flows in ordinary years.

12.—In isolated basins with no surface stream outlet the problem is less complicated by traditional concepts of what constitutes a water right. The average recharge and draft can be estimated with reasonable accuracy after thorough study. At first thought, it would seem that a simple solution would be to issue permits up to the amount of recharge as a greater draft which may be sustained for a period would only necessitate a future decrease. However, decrease in draft most often occurs because cost of pumping from a lowered water-table becomes too great for some users and not because the water has become physically unavailable. The recharge during a long period of years may be deficient due to vagaries of the climate. Even though pumping draft is less than the long-time average recharge the water-table will drop during such periods, and excessive pumping costs will occur so that some users quit. The result is different only in degree from that which would be the case with actual long-time overdraft. Obviously, the administrator is faced with consideration of water costs to determine the safe yield in such situations and this, instead of merely quantity of water available, may guide his decision.

13.—The administrator will also be faced with another situation not discussed in the body of this paper and pertinent only to the economic questions which are potentially present in the intensive use of almost all streams, but particularly so in the intensive use of underground water. The use and re-use of water increase the content of dissolved solids. Some of the salts held in suspension appear to be actively poisonous to plant growth whereas others merely decrease the growth by partial smothering. Still others destroy the soil and make it unworkable. Theoretically, the use of water from an underground basin can be developed to equal the average recharge, but this would result in a slowly cumulative concentration of solids possibly to the point at which all agricultural values would be destroyed. A certain percentage of the supply must be wasted to carry off the salts. This fact must be considered in any intelligent administrative control of underground water.

14.—The foregoing sections refer to natural underground supply. When water is diverted from a stream it passes into the possession of the appropriator. As more water than can be consumed is almost always diverted and placed on the land the surplus percolates to the water-table, and, unless pumped out, finally reaches the stream. The law differs between States; in some States the water which has percolated, is deemed the property of the diverter until it reaches a natural channel, and hence it can be pumped and used on additional land under his control without interference with other users down stream. In other States the diversion is deemed to be applicable for use on the land which it will serve directly. The use is deemed completed when the water in the surface conduit is exhausted, and deep percolation is regarded as belonging to the stream system. In such cases, it cannot be pumped without a permit, which would not be given in most cases for reasons previously stated.

15.—The decisions of the Idaho Court (Silkey *v.* Trego, 5 P. 2d, 1049, 1931; and Noli *v.* Stonen, 26 P., 1112, 1933) prohibited additional pumping from an underground basin because it lowered the water-table and thereby increased the pumping cost for a prior user. The decisions of the Arizona Court cited previously follow the same reasoning, but in this case the underground water is regarded as being in a definite channel, although several miles wide. As underground water cannot be used without lowering the water-table, this reasoning, if persisted in, means inhibition of the further development of underground water in these jurisdictions. The fact that considerable underground water is used in these States is no criterion of the future. This use has created a body of water users who will unite behind these decisions to enjoin additional use.

16.—In any State having the appropriative doctrine it should not be difficult to enact a statute that will clearly put underground water under the control of the water authority, or to frame rules and regulations in those States in which the status is already clear. If the doctrine of the statute is compatible with the appropriative doctrine it clearly means prohibition of further underground-water development when the water authority takes control of an underground basin which contributes to a surface stream for reasons before stated.

In States recognizing a different administration of water law than the appropriative doctrine, statutes regulating the use of underground water, if enacted, must rest on police power. In a State wherein the water law is as well developed as is the case in California, such a statute would probably extend only to prohibition in the case of overdraft and would not have the effect noted in the preceding paragraph.

17.—Lack of administrative control leads to the fullest development of underground water but, at the same time, it leads to excess development. A statute creating control, such as those adopted by New Mexico, Oregon, and Utah, apparently permits the water authority to declare control at his option. This permits a thorough investigation of the situation prior to taking control and is a flexible arrangement which may guard against the evils of too much and too little control.

18.—Perhaps future development, particularly in cases where the regulation of underground water is involved, will bring sharply to the fore a trend toward modification of water laws applicable to smaller units than those involved in such agreements as the Colorado River Compact.

19.—To summarize: Administrative control of underground water, to be intelligent, requires much research and investigation, which, in turn, requires large legislative appropriations. When administrative control is initiated it may well result in the prohibition of underground water development in States following the appropriative doctrine, in most basins tributary to surface streams, and plunge the administrator into the question of economics in isolated basins. On the other hand, control will stop over-development and thus becomes desirable at some stage no matter what the legal doctrine of the jurisdiction. If the fullest development of the water resources of a State are desired, administrative control of underground water should

not be adopted until overdraft threatens, and then only in particular areas. Conditions probably would never warrant administrative control in the arid States having a cool climate or in most of the humid States.

20.—All of the foregoing comments leads to a re-statement of the paramount conclusion that, as control of underground water becomes necessary in the various jurisdictions, the complexities of the situation can be dealt with only by lodging greater power in the water authority than is now given in most jurisdictions. However, a step at a time may be the safest course.

Acknowledgments

The writer expresses his appreciation of the authorities cited in this paper and acknowledges his debt in the preparation of the résumé of the law given herein. In addition, he expresses his appreciation of the corrections and criticisms offered by the members of the Committee of the Irrigation Division on the Conservation of Water.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE SHEAR-AREA METHOD

Discussion

By Messrs. F. Muls-Guinotte, and Horace B. Compton and Clayton O. Dohrenwend

F. Muls-Guinotte,[30] Esq. (by letter).[30a]—The well-known theorem which states that the shear diagram may be obtained by the successive derivatives of the elastic curve, is the basis of this paper. The theory of beams on elastic foundations is based on the principle that a fourth derivative of the elastic curve gives the loading diagram. The authors give the value of the successive derivatives in Fig. 1 of the paper, making the implicit assumption that the moment of inertia is constant. This assumption is not necessary. It is quite easy to obtain the elastic curve starting from the loading diagram when the rule of successive derivatives is written, as follows: $\frac{dw}{dx} = \phi_x$; $\frac{d\phi_x}{dx} = \frac{M_x}{E\,I_x}$ $\left(\text{or } M_x = E\,I_x\,\frac{d\phi_x}{dx}\right)$; $\frac{dM_x}{dx} = T_x$; and, $\frac{dT_x}{dx} = p_x$.

This suggestion for developing the p_x-diagram indicates how the equation of the elastic curve may be solved in the general case of a variable moment of inertia.

The variable loading of a tapered beam is shown in Fig. 48(a); the first integration yields the shear diagram (Fig. 48(b)); and, the second integration results in the moment diagram (Fig. 48(c)). If the latter curve is divided by the product, $E\,I$, at successive ordinates the resulting curve represents the M-function (Fig. 48(d)) which, when integrated, produces, successively, the slope curve (Fig. 48(e)) and the deflection curve (Fig. 48(f)).

It is also of interest to show the results of deriving the diagram of moments when the elastic beam is a curved one. Suppose, first, that there are no forces

Note.—The paper by Horace B. Compton, Assoc. M. Am. Soc. C. E., and Clayton O. Dohrenwend, Jun. Am. Soc. C. E., was published in May, 1935, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: August, 1935, by Messrs. George E. Large, Samuel T. Carpenter, Roland H. Trathen, A. W. Fischer, J. Charles Rathbun, Harold R. Kepner, and Fred L. Plummer; October, 1935, by Messrs. Albin H. Beyer, John M. Beatty, R. B. Peck, Ralph W. Stewart, C. W. Johnson and H. W. Birkeland, Garrett B. Drummond, and Harold E. Wessman; and December, 1935, by Messrs. Fang-Yin Tsai and David A. Molitor.

[30] Asst. in Univ. of Liége, Liége, Belgium.

[30a] Received by the Secretary January 7, 1936.

applied on the element, ds (Fig. 49), which is then kept in equilibrium by the forces M, T, N, $M + d\,M$, $T + d\,T$, and $N + d\,N$. For the condition, $\Sigma\, M = 0$:

$$d\,M + T\,\Delta\,s\,\cos\Delta\,\theta + N\,\Delta\,s\,\sin\Delta\,\theta = 0$$

for $\Sigma\, N = 0$:

$$N - (N + d\,N)\cos\Delta\,\theta - (T + d\,T)\sin\Delta\,\theta = 0$$

and, for $\Sigma\, T = 0$:

$$T - (T + d\,T)\cos\Delta\,\theta + (N + d\,N)\sin\Delta\,\theta = 0$$

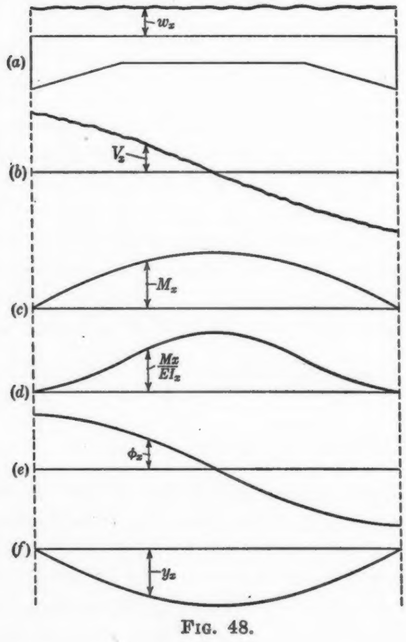


Fig. 48.

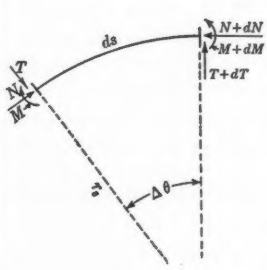


Fig. 49.

Since the element, ds, is very small: $\cos\Delta\,\theta = 1$; $\sin\Delta\,\theta = \Delta\,\theta$; and, $\Delta\,\theta = \dfrac{\Delta s}{r_s}$. When ds tends to become zero; $\dfrac{dM}{ds} = T_s$; $\dfrac{dT}{ds} = +\dfrac{N}{r_s}$; and, $\dfrac{d\,N}{ds} = -\dfrac{T}{r_s}$.

The coefficient, $\dfrac{1}{r_s}$, may be eliminated by using a derivative with respect to the variable, θ, instead of the variable, s. With this elementary change: $\dfrac{d\,N}{d\theta} = -T$; and $\dfrac{dT}{d\theta} = N$. Consequently, it is possible to define a rule by

which the moment curve can be derived successively. Assuming that $T = \psi_s$:

$$\frac{dM}{ds} = \psi_s; \frac{d\psi_s}{d\theta} = N; \frac{d^2\psi_s}{d\theta^2} = -T; \frac{d^3\psi_s}{d\theta^3} = -N; \frac{d^4\psi_s}{d\theta^4} = T; \frac{d^5\psi_s}{d\theta^5} = N; \text{etc.},$$

and $\dfrac{d^{4n}\psi_s}{d\theta^{4n}} = T; \dfrac{d^{4n+1}\psi_s}{d\theta^{4n+1}} = N$; etc.

The foregoing properties of the successive derivatives may yield results of particular interest in the study of influence lines.

When external forces occur on the section of beam analyzed the components along the normal or the tangent appear in the equations of equilibrium. These components are easily introduced, and, in the case of radial forces, special and simple results are obtained.

HORACE B. COMPTON,[31] ASSOC. M. AM. SOC. C. E., AND CLAYTON O. DOHRENWEND,[32] JUN. AM. SOC. C. E. (by letter).[32a]—The shear-area method was presented to enlarge on the usual methods for the solution of slopes and deflections in beams. It was stated that the method was particularly adapted to those problems which involve distributed loads (when compared with solutions by the moment-area method) and still would solve other problems readily. Solutions which consider variation in the moment of inertia of the beam sections are probably obtained with a greater degree of safety by the moment-area method.

Since it appears that the engineer remembers the procedure for the solution by the conjugate beam for slopes and deflections better than by the slope-deviation method, the mathematical beam was introduced.

In most cases the tendency of the commentators has been to follow the very well known and common solutions of moment area and not to discuss the subject in question. Discussions of simple problems by the moment-area method need no further comment. However, in contrast, Professor Kepner with a few others has added materially to the subject.

The end slopes of the shaft indicated by Mr. Fischer may be solved by writing deflection equations about the ends of the shaft; thus, about the right end, $y = 0 = \phi_L\, L - \frac{1}{2}\, I_L$ by Equation (7). Therefore:

$$E\,\phi_L\,60 - \frac{1}{2}\Big[56.5 \times \frac{23^3}{12} + 56.5 \times 23 \times 48.5^2 + 14.71 \times \frac{5^3}{12} - 961.2 \times 37^2$$
$$+ 14.71 \times 5 \times 34.5^2 - 2.27 \times \frac{12^3}{12} - 2.27 \times 12 \times 26^2 + 1\,072 \times 15^2$$
$$- 19.24 \times \frac{5^3}{12} - 19.24 \times 5 \times 17.5^2 - 90.68 \times \frac{15^3}{3}\Big] = 0$$

and $\phi_L = \dfrac{16\,474}{E}$. Mr. Fischer has indicated a similar procedure by letter. This method shortens the problem by the shear-area method considerably. The difficulty of Mr. Peck's problem can be overcome in the same manner.

[31] Asst.. Prof. of Mechanics, Rensselaer Polytechnic Inst., Troy, N. Y.
[32] Instr., Dept. of Civ. Eng., Rensselaer Polytechnic Inst., Troy, N. Y.
[32a] Received by the Secretary January 15, 1936.

The writers wish to express their appreciation for the constructive criticism offered by Professor Kepner, especially for demonstrating, by the use of Equation (7), a short method for obtaining end slopes. In correspondence submitted directly to the writers, Mr. Stewart suggested the following for the problem solved by the conjugate beam method in Mr. Beatty's discussion:

$$\phi_L = \frac{1}{E\,I}\,[2\,400 \times 12 + 1\,800 \times 8]\;144 = \frac{6\,220\,800}{E\,I}$$

It is evident that the solution for the reaction can be given in a somewhat simpler manner than that indicated by Mr. Beatty by considering the area of *BCDGB* (Fig. 40(c)) as two-thirds of a rectangle with dimensions, *BG* and *GD*.

In the discussion by Messrs. Johnson and Birkeland it is inferred that there is no difference in the work necessary to solve the first moment of the moment area, or the second moment of the shear area, because in the two cases the integrations are comparable. This inference is not supportable except in deriving the two cases, since the use of the cases is not by integration directly, but by replacing the integration with equivalent expressions. The mathematical demonstrations given, showing the relations between the functions is very complete, but the following formula, due to Lejeune Dirichlet:

$$\int_a^b dx \int_{y_2}^{y_1} f\,(x,\,y)\;dy = \int_c^d dy \int_{x_1}^{x_2} f\,(x,\,y)\;dx$$

necessary in the solutions shown, is not common enough to engineers to use without some explanation. If more emphasis had been given to specific cases the following statement would have been considerably modified: "The method does not seem to be readily adaptable to any but the most simple beam problems such as statically determinate, constant section cases with uniformly distributed load."

Most of the apparent difficulties encountered by Professor Tsai will be clarified if it is recalled that the mathematical beam is hypothetical and has slope with no deflection. The loading of the mathematical beam is the shear diagram; the first moment yields the slope of the given beam only when divided by $E\,I$, and the second moment or moment of inertia gives deflection of the original beam only when divided by $2\,E\,I$. This means that for the two cases mentioned, the diagram on the beam can be considered as the shear divided by $E\,I$.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FAILURE THEORIES OF MATERIALS SUBJECTED TO COMBINED STRESSES

Discussion

BY JOSEPH MARIN, JUN. AM. SOC. C. E.

JOSEPH MARIN,[30] JUN. AM. SOC. C. E. (by letter).[30a]—The interpretation of the test results for materials subjected to combined stresses as given by Professor Slade offers an interesting method of attack. It is essentially the same as the method used in the modified Mohr theory (Theory (9)), since in both cases an average value of the test results is used. As might be expected, both these semi-empirical methods give approximately the same results. It is apparent that Professor Slade's interpretation is more flexible and basic in its application. However, the usefulness of his method in this particular problem, is reduced in view of the new data presented in this closure.

Mr. Jasper has revealed a significant factor in referring to the error introduced in some experiments due to a buckling type of failure. This danger, however, can be avoided by a preliminary analysis in which the size of the specimen can be determined in such a manner that failure by buckling is avoided.

The writer wishes to thank Mr. Silverman for referring to the work of Lode and that of Ros and Eichinger. Later work by these investigators has been reported.[31] In addition, the recent experimental work of Taylor and Quinney[32] should be mentioned. A further study by the writer, of the experimental results obtained by these investigators and others, shows that for ductile metals the maximum shear energy theory (Theory (11)) gives decidedly the best agreement with the experimental results, whereas, for a brittle mate-

NOTE.—The paper by Joseph Marin, Jun. Am. Soc. C. E., was published in August, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1935, by Messrs. J. J. Slade, Jr., T. McLean Jasper, and I. K. Silverman; November, 1935, by Messrs. W. P. Roop, and H. F. Moore; and December, 1935, by Messrs. A. A. Eremin, and A. Floris.

[30] Asst. Prof., Eng. Materials, Dept. of Gen. Eng., Rutgers Univ., New Brunswick, N. J.

[30a] Received by the Secretary February 24, 1936.

[31] "Der Einfluss der mittleren Haupspannung auf das Fliessen der Metalle", by W. Lode. V. D. I. *Forshungsarbeiten*, Heft 303, 1928; also, "Eigenössische Materialprüfungsanstalt an der E. T. H. in Zurich, Versuche zur Klarung der Frage der Bruchgefahr", by M. Ros and A. Eichinger, *Diskussionsbericht*, Nr. 28 (1928) and Nr. 34 (1929).

[32] "Plastic Distortion of Metals", by G. I. Taylor and H. Quinney, *Philosophical Transactions*, Royal Soc. of London, Series A, Vol. 23, 1931, p. 323.

rial, such as cast iron, the few test results available show that the maximum stress theory agrees best with the experiment. The basis for the formulation of the maximum volume energy theory as requested by Mr. Silverman, is that failure in the case of an element subjected to combined stresses is assumed when these stresses have reached such values that the volume change produced as a result of deformations reaches the value of the volume change produced in the case of simple tension at failure.

The writer does not understand the statement made by Lieut. Comdr. Roop that "the need for a 'failure theory' is not immediately apparent." The ultimate strengths in simple stress are not always denoted by the points, $x = \pm 1$, $y = \pm 1$. It is the practice usually to define failure for a ductile material such as steel by the lower yield point (if such exists), or a proof stress, while for a brittle material such as cast iron it is the practice to use the ultimate stress. The more definite conclusion requested by Lieut. Comdr. Roop is given in the foregoing paragraphs.

Professor Moore mentions the tests on cast iron subjected to combined stresses made by Matsumura and Hamabe as supporting the maximum strain theory. However, tests on cast iron subjected to combined stresses made by Ros and Eichinger[31] and others made by Cook and Robertson[33] show that the maximum stress theory is in closer agreement with the test results. Regarding fluctuating stresses, tests by Gough[34] more recent than those mentioned by Professor Moore verify that, for the two kinds of steel tested, the maximum shear energy theory is a good approximation. The writer agrees with Professor Moore that there is sufficient evidence to show that different theories hold for ductile and brittle materials. Some recent work has been done on the subject of creep under combined stresses, both experimentally and analytically, principally by Bailey.[35]

The problem of the failure of materials subjected to tri-axial static stresses was intentionally omitted from the paper in order to present first the simplest case. In answer to Professor Moore's comment about tri-axial stresses, the researches of von Kármán[36] and Boker[37] on brittle materials, and more recently that of Cook[38] on steel, should be mentioned. There are apparently many problems yet to be investigated regarding the failure of materials subjected to static and fluctuating combined stresses at both normal and elevated temperatures.

---

[33] "The Strength of Thick Hollow Cylinders Under Internal Pressure", by G. Cook and T. Robertson, *Engineering*, Vol. 92, p. 786.

[34] Rept. of National Physical Laboratory, England, by H. C. Gough, *Engineering*, July 12, 1935, p. 43.

[35] "Creep of Steel Under Simple and Compound Stresses", by R. W. Bailey, *Engineering*, January–June, 1930, Vol. 129, pp. 265, 327; also, "Utilization of Creep Test Data in Design", *Engineering*, December, 1935, and preprint for November meeting of the Inst. of Mech. Engrs., London, England.

[36] "Festigkeitsversuche unter allseitigem Druck", Th. von Kármán, M. Am. Soc. C. E., *Mitteilungenüber Forschungsarbeiten*, V. D. I., Heft 118, 1912.

[37] "Die Mechanik der bleibenden: Formänderung in Kristallinischaufgebauten Körpern", by R. Boker, *Mitteilungenüber Forschungsarbeiten*, V. D. I., Heft 175–176, 1915.

[38] *Proceedings*, Royal Soc. of London, 1931, Vol. 37, p. 559.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ADAPTATION OF VENTURI FLUMES TO FLOW MEASUREMENTS IN CONDUITS

Discussion

BY PROF. ING. FILIPPO ARREDI

PROF. ING. FILIPPO ARREDI[24] (by letter).[24a]—The sole purpose of this discussion is to suggest a variation in the authors' method of calculating discharge for the given case, which is a little different from that described in the paper and, possibly, slightly more practical.

Equations (8) and (10) of the paper apply to the throat section, if the water is assumed to flow at critical velocity. For the section of the channel above the throat Equation (2) applies, in which ϵ and Q have the same values as in the throat, and d and A are the depth of water and the area of the wetted section of the channel, respectively.

The method of calculating the Venturi flume which the authors propose is as follows: Taking a value for d_c, the values of A and B are calculated for the throat, and the values of Q and ϵ are then obtained from Equations (8) and (10). Then, from Equation (2), substituting these values of Q and ϵ, d can be obtained by trial, taking account of the relations between A and d.

As it will usually be necessary to study throats of various types in order to make a choice among them, it will also be necessary to make calculations for many values of Q, and considerable work, requiring a large amount of time, may be needed. For this reason, it may perhaps be useful to supplement the authors' method by a graphical device which may prove more rapid in that it does not require "trial-and-error." The relation,

$$\frac{V^2}{2\,g} = \frac{Q^2}{2\,g}\,\frac{1}{A^2} \qquad (48)$$

applies for both the channel and the throat. If a value is assumed for $\frac{Q^2}{2\,g}$

NOTE.—The paper by Harold K. Palmer, M. Am. Soc. C. E., and Fred D. Bowlus, Assoc. M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. N. F. Hopkins, and Hunter Rouse; and February, 1936, by Messrs. F. V. A. E. Engel, and J. C. Stevens.

[24] Asst., Cattedra di Costruzioni Idrauliche, Facoltà d'Ingegneria, R. Univ. di Roma, Rome, Italy.

[24a] Received by the Secretary December 30, 1935.

and arbitrary values are taken for A, the corresponding values of $\frac{V^2}{2\,g}$ may be obtained easily from Equation (48). The computation is repeated for various values of $\frac{Q^2}{2\,g}$. Curves corresponding to each value assumed for $\frac{Q^2}{2\,g}$ are then drawn in a diagram (see Fig. 20) in which the values of $\frac{V^2}{2\,g}$ are ordinates and those of A are abscissas.

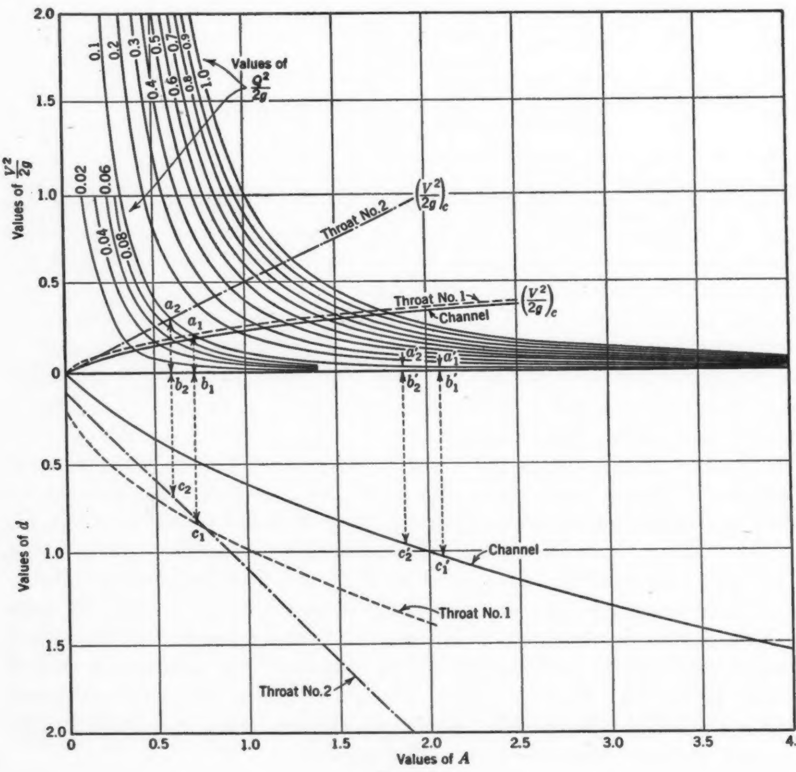


FIG. 20.

Knowing the section of the canal, and assuming the sections of one or more throats, the relation between the values of A and the depth, d, becomes known for each case, d being taken with respect to a common datum. In Fig. 20 the curve that expresses A in terms of d, for either channel or throat, is drawn downward from the zero line.

For critical conditions, the relation,

$$\left(\frac{V^2}{2\,g}\right)_c = \frac{A}{2\,B} \qquad \text{.........................(49)}$$

applies. Assuming values of d_c in the throat, corresponding values of A and B are calculated, and $\left(\frac{V^2}{2g}\right)_c$ is computed by means of Equation (49), and the curve showing $\left(\frac{V^2}{2\,g}\right)_c$ as a function of A can be drawn intersecting the curves already drawn. It may be useful, although not necessary, to draw a similar curve for the channel also (see Fig. 20).

Assuming, for example, that a discharge, Q, such that $\frac{Q^2}{2\,g} = 0.1$, flows through Throat No. 1 in the critical condition, it is then easy to see that the ordinate, $a_1\,b_1$, represents the velocity head. In Fig. 20, a_1 is the point of intersection of the $\frac{V^2}{2\,g}$ - curve, plotted as a function of A (for $\frac{Q^2}{2\,g} = 0.1$), with the curve of $\left(\frac{V^2}{2\,g}\right)_c$ for Throat No. 1. The dimension, $b_1\,c_1$, represents the depth of water, c_1 being the intersection of the vertical through a_1, with the A-curve plotted in terms of d for Throat No. 1; and the length, $a_1\,c_1$, being the total energy head in the throat.

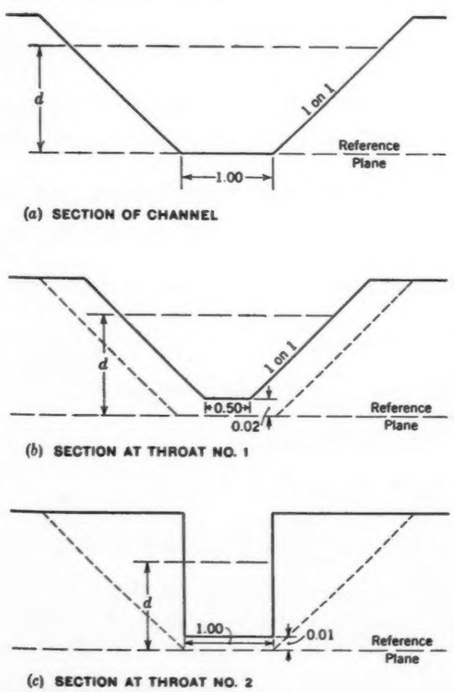


FIG. 21.

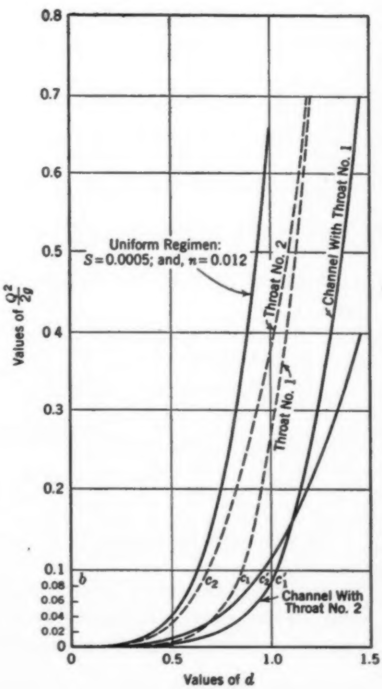


FIG. 22.

The next step is then to find a vertical segment, $a'_1\ c'_1$, which will be equal in length to $a_1\ c_1$ and which will have its upper end on the $\frac{V^2}{2\ g}$- curve in terms of A for $\frac{Q^2}{2\ g} = 0.1$, and its lower end on the d-curve in terms of A for the channel. Then, $a'_1\ b'_1$ will be the velocity head, $b'_1\ c'_1$, the depth of water, and $a'_1\ c'_1$, the total energy head above the throat. In a similar manner, it is seen that the segment, $b_2\ c_2$, represents the depth of water in the throat and $b'_2\ c'_2$, the depth of water in the channel, if the same discharge flows through Throat No. 2 in the critical condition, $a'_2\ c'_2$ being equal to $a_2\ c_2$.

It is evident that the curve of $\frac{V^2}{2\ g} = f(A)$ for any constant, $\frac{Q^2}{2\ g}$, will be the same, whatever the channels and throats may be. The other curves in Fig. 20 were drawn by assuming channels and throats such as those in Fig. 21. The depth, d, is always referred to the plane through the channel bottom, at a given section. The detailed formulas by which these curves were computed are not reported.

In order, rapidly, to sketch the profiles of the water surface in the flume, it will be useful to make diagrams such as those in Fig. 22, in which the values of $\frac{Q^2}{2\ g}$ are entered as ordinates and the values of d (computed as described previously), as abscissas. The curves of $\frac{Q^2}{2\ g}$, as a function of d, are drawn for the throats and the channel above the throat. Then, in Fig. 22, at $\frac{Q^2}{2\ g} = 0.1$, are recorded the ordinates, bc_1, bc_2, bc'_1, bc'_2, obtained from Fig. 20, which represent the depth, d, for Throats Nos. 1 and 2 and for the water in the channel above either Throat No. 1 or Throat No. 2. By varying $\frac{Q^2}{2\ g}$, it becomes possible to obtain a sufficient number of points to trace such curves.

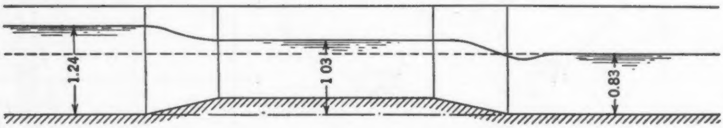


Fig. 23.—Profile, Throat No. 1; $\frac{Q_2}{2\ g} = 0.3$.

It will also be useful to draw the curve of $\frac{Q^2}{2\ g}$ as a function of d for the uniform regimen that is present below the throat. In Fig. 22, this curve has been drawn on the basis of Manning's formula, assuming a slope, $S = 0.0005$ and $n = 0.012$. Fig. 23 represents a profile with Throat No. 1, relative to a discharge, Q, such that $\frac{Q^2}{2\ g} = 0.3$, the values for d of 1.24 above the throat, 1.03 at the throat, and 0.83 below the throat having been obtained from Fig. 22.

By the same method by which the curves of d were drawn in Fig. 22, it is easy to draw those for ϵ as a function of the $\frac{Q^2}{2\,g}$- curves which may be considered equal to those drawn by the authors in Figs. 8 and 10. It would then be easy to select from the throats studied the one best adapted to a particular case, more satisfactorily than under the conditions recorded by the authors in their paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE STRESS FUNCTION AND PHOTO-ELASTICITY APPLIED TO DAMS

Discussion

By D. P. Krynine, M. Am. Soc. C. E.

D. P. Krynine,[36] M. Am. Soc. C. E. (by letter).[36a]—The use of the Airy stress function in dam engineering is discussed in this highly interesting and important paper. The writer's attention has been especially attracted by the determination of stresses in foundations (Application II).

Limits of Application of the Stress Function to Dam Foundations.—As stated in the "Introduction" the use of the stress function is restricted to elastic isotropic materials. Granite or basalt rock may be considered as elastic and, to a certain extent, isotropic; and, consequently, elastic theories may be applied in the case of such foundations. The situation is more complicated if the foundation material is sandstone, for example, since it is questionable whether in all cases such a material satisfies the conditions of both elasticity and isotropy. On the other hand, a clay foundation would justify the application of the stress function, whereas, in the case of glacial drift, such an application would be misleading since the actual stress distribution is far different from the patterns given by the theory of elasticity.

The use of the stress function is always limited to the two-dimensional problem. In other words, satisfactory results may be expected from the application of the stress function to sections at some distance from the ends of the dam, but not otherwise. Near the ends, the problem is three-dimensional, and the stress function is not applicable.

Within these limitations Mr. Brahtz has demonstrated, in a comprehensive and intelligent manner, that stresses in a dam foundation can be determined by means of the stress function. It still remains to be proved that they should be thus determined and the writer is rather pessimistic in this respect.

Note.—The paper by John H. A. Brahtz, Esq., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.; December, 1935, by Fred L. Plummer, Assoc. M. Am. Soc. C. E.; January, 1936, by Messrs A. G Solakian and Lars R. Jorgensen; and February 1936, by Elmer O. Bergman, Assoc. M. Am. Soc. C. E.

[36] Research Assoc. in Soil Mechanics, School of Eng., Yale Univ., New Haven, Conn.

[36a] Received by the Secretary February 15, 1936.

Stress Distribution Under a Foundation.—It can be demonstrated that the value of the stress, σ, caused by a concentrated force, P, acting at the boundary of a semi-infinite earth mass (Fig. 16), is:

$$\sigma = \frac{\nu\, P}{2\,\pi\,\rho^2} \sin^{\nu-2}\theta \qquad (99)$$

Equation (99) has been proposed by John H. Griffith[37], M. Am. Soc. C. E.,

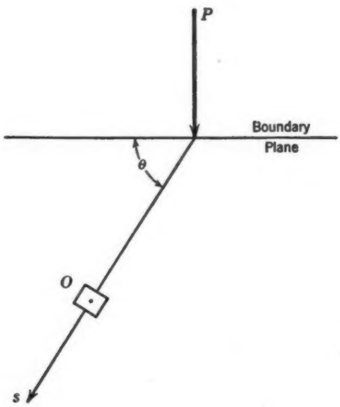


FIG. 16.

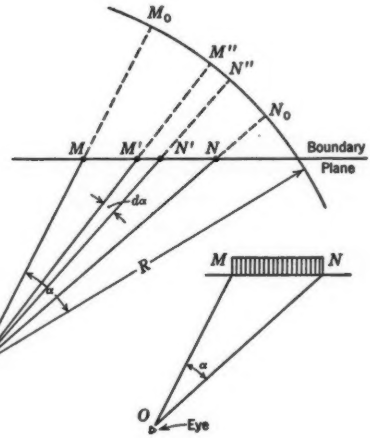


FIG. 17.

and Dr. O. K. Fröhlich.[37] It can be developed without making use of elastic theories. Hence, it is valid for any isotropic or statistically isotropic body. The coefficient, ν, is the so-called "concentration factor", and is equal to 3, for the case of an isotropic elastic body in which the value of the reciprocal of Poisson's ratio, m, is equal to 2. For this case Equation (99) becomes:

$$\sigma = \frac{3\,P}{2\,\pi\,\rho^2} \sin\theta \qquad (100)$$

Equation (100) may be obtained from those developed by Boussinesq[6], assuming that $m = 2$. A two-dimensional analogue to Equation (100) is Michell's expression for radial distribution[38]:

$$\sigma = \frac{2\,P}{\pi\,\rho} \sin\theta \qquad (101)$$

The plane stress distribution does not depend on the value of Poisson's ratio, μ, and Equation (101) holds in the case of any value of that ratio.

Concentrated Forces at the Boundary of a Mass.—The problem in connection with Fig. 3 of the paper, in the case of a two-dimensional elastic body,

[37] "Pressures Under Substructures", by John H. Griffith, M. Am. Soc. C. E., *Engineering and Contracting*, March, 1929, pp. 113–119; also, "Drukverdieling in Bouwgrond", by Dr. O. K. Fröhlich, *De Ingenieur*, April 15, 1932, p. B–52.

[6] "Applications des Potentiels à l'Etude de l'Equilibre et du Mouvement des Solides Elastiques", by J. Boussinesq, Paris, 1885.

[38] "Theory of Elasticity", by S. Timoshenko, p. 82, 1934.

may be treated in the form of a general case (that is, for any three-dimensional body, isotropic or statistically isotropic), by applying Equation (99). In the case of a horizontal force, Q, the solution may be written directly[39]:

$$\sigma_x = -\frac{\nu Q}{2\pi\rho^2}\cos^\nu\theta \quad \text{.................(102a)}$$

$$\sigma_y = -\frac{\nu Q}{2\pi\rho^2}\cos^{\nu-2}\theta\sin^2\theta \quad \text{...........(102b)}$$

and,

$$\tau_{x,y} = -\frac{\nu Q}{2\pi\rho^2}\cos^{\nu-1}\theta\sin\theta \quad \text{............(102c)}$$

A two-dimensional analogue to Equations (102) for the particular case in which $\nu = 3$ leads directly to Equations (32) of the paper.

In the same manner the stresses caused by a vertical force, P, acting at the boundary are:

$$\sigma_x = -\frac{\nu P}{2\pi\rho^2}\sin^{\nu-2}\theta\cos^2\theta \quad \text{..............(103a)}$$

$$\sigma_y = -\frac{\nu P}{2\pi\rho^2}\sin^\nu\theta \quad \text{.....................(103b)}$$

and,

$$\tau_{x,y} = -\frac{\nu P}{2\pi\rho^2}\sin^{\nu-1}\theta\cos\theta \quad \text{................(103c)}$$

For the case of a two-dimensional elastic mass the three-dimensional formulas (Equations (103)) may be easily reduced to those given by Mr. Brahtz immediately preceding Equation (37).

Uniform Load Distribution at the Boundary of a Two-Dimensional Mass; "Angle of Visibility".—The problem of a uniformly distributed load acting at the horizontal boundary of a mass may be also solved by applying the general formula, Equation (99). For the purposes of this discussion, a two-dimensional elastic mass only will be dealt with and Equation (101) will be applied. In Equations (38) and (39) Mr. Brahtz gives the values of the stresses, σ_x, σ_y, and $\tau_{x,y}$, as functions of two angles, α_1 and α_2. It is more convenient to use a single angle, $\alpha = \alpha_1 + \alpha_2$ (see Fig. 17). The angle, α, may be termed the "angle of visibility" since this is the angle at which a hypothetical observer placed at the given point, O, of the mass, would see the foundation if the mass were transparent. The writer understands that the conception of the "angle of visibility" was first introduced by Professor N. M. Gersevanov.[40] Tracing a circle of an arbitrary radius, R, with Point O as a center, and considering an elementary angle of visibility, $d\alpha$, corresponding

[39] "Druckverteilung im Baugrunde", by O. K. Fröhlich, p. 27, 1934.

[40] "Principles of Dynamics of an Earth Mass", by N. M. Gersevanov, p. 153 (printed in Russian, in 1933).

to a loaded element, $M' N'$, at the boundary, the elementary stress, $d\sigma$, at Point O, would be, using Equation (101):

$$d\sigma = -2p \left[\frac{\rho \, d\alpha}{\cos \theta} \right] \frac{1}{\pi \rho} \cos \theta = - \frac{p}{\pi} \, d\alpha \qquad (104)$$

Were the mass cylindrical (that is, circular in cross-section) with a radius, R, and were the arc, $M_0 N_0$, loaded normally with the unit load, p, the corresponding elementary stress, $d\sigma$, at Point O, would be:

$$d\sigma = -2p \, [R \, d\alpha] \frac{1}{\pi R} = - \frac{p}{\pi} \, d\alpha \qquad (105)$$

since in this case the force, $p \, [R \, d\alpha]$, acts normally to the circular boundary of the mass, and the angle, θ, equals zero. Integrating Equations (104) and (105) through the entire angle, α, it may be stated that the sum of the principal stress, σ, at Point O, is proportional to the "angle of visibility", α:

$$\sigma = - \frac{2p}{\pi} \alpha \qquad (106)$$

It is also obvious that the sum of principal stresses at a point does not depend on the shape of the loaded surface provided the "angle of visibility", α, of the loaded surface remains the same, and the surface in question is loaded uniformly and normally.

It may be seen from Fig. 17 that using the "angle of visibility" an asymmetrical loading problem (loaded surface, $M N$) is reduced to a symmetrical problem (loaded surface, $M_0 N_0$). This fact permits one to state directly that the direction of the major principal stress coincides with the bisector of the "angle of visibility." It is well known that the proof of this simple statement, as given in textbooks on elasticity, is rather complicated.

Projecting all the elementary stresses, $d\sigma$, on the normal to a plane, and integrating, the pressure normal to that plane is obtained. Thus:

$$d\sigma_x = - \frac{2p}{\pi} \, d\alpha \cos^2 \theta = - \frac{p}{\frac{1}{2} \pi R^2} \, [R \, d\alpha \cos \theta] \, [R \cos \theta] \qquad (107a)$$

and,

$$d\sigma_y = - \frac{2p}{\pi} \, d\alpha \sin^2 \theta = - \frac{p}{\frac{1}{2} \pi R^2} \, [R \, d\alpha \sin \theta] \, [R \sin \theta] \qquad (107b)$$

It may be seen from Fig. 18(a) that $[R \, d\alpha \sin \theta]$ and $[R \, d\alpha \cos \theta]$ are projections of the arc element, $R \, d\alpha$, on horizontal and vertical planes, respectively. In the same manner $[R \cos \theta]$ *and* $[R \sin \theta]$ are projections of the radius, R (also on horizontal and vertical planes, respectively). Designating the half area of a circle of a radius, R, by A, and integrating (Fig. 18(b)):

$$\sigma_x = - p \, \frac{\text{Area } M_0 \, M''_0 \, N''_0 \, N_0}{A} \qquad (108a)$$

and,

$$\sigma_y = - p \, \frac{\text{Area } M_0 \, M'_0 \, N'_0 \, N_0}{A} \qquad (108b)$$

Projecting now the arc, $M_0 N_0$, on the normal to the bisector of the "angle of visibility", the values of the principal stresses, σ_1 and σ_2, may be obtained. The minor principal stress, σ_2, is proportional to the area between the chord,

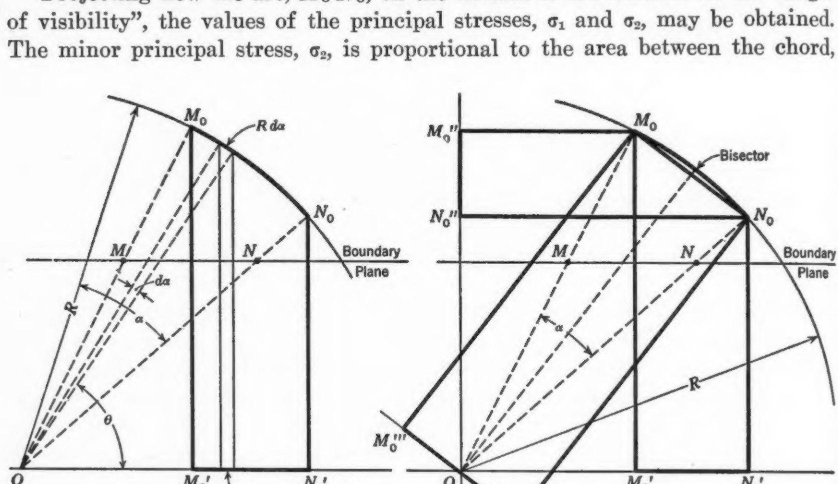


FIG. 18.

$M_0 N_0$, and the arc, $M_0 N_0$. The major principal stress is proportional to the sum of that area and the rectangle, $M_0 M'''_0 N'''_0 N_0$. Shearing stresses may be easily obtained in the same way. Thus, the determination of stresses is reduced to a simple measurement of areas by planimeter. This method may be extended to the case of non-uniformly distributed vertical loads, which represents a problem that is almost impossible to attack by the use of the

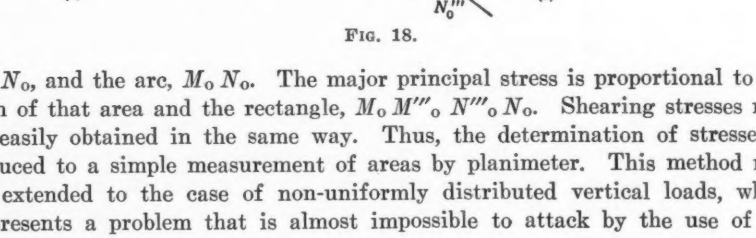


FIG. 19.

stress function method. Fig. 19(*a*) illustrates the determination of the stress, σ_y, in such a case. Each ordinate of the area, $M_0 M'_0 N'_0 N_0$, is to be multiplied by the ratio, $\frac{p_1}{p}$, of the actual unit load, p_1, at a given point of the foundation, to a certain standard unit load, p.

A horizontal force, Q, and a vertical, $P = Q \cot \theta$, produce equal stresses at a given point which follows from a simple inspection of the formulas erpressing the stresses, σ_x, σ_y, and $\tau_{x,y}$. The problem of uniform distribution of a horizontal force, q_1, per unit of length of the distance, l_1, may be thus reduced to that of a non-uniform distribution of the vertical unit force, $q_1 \cot \theta$, over the same distance, l_1. A straight line is drawn with its origin, O', at the boundary of the mass (Fig. 19(*b*)). The slope of this line is $\frac{q_1}{y}$, in which y is the depth of the given point, O. The "angle of visibility", α, would determine an area, $M M' N' N$, showing the law of distribution of the equivalent vertical load over the distance, $M N = l_1$. The problem is afterward solved according to Fig. 19 thus avoiding Equations (34). The problem of a non-uniform distribution of a horizontal force, q_1, may be also solved in a similar manner.

Reservoir Pressure.—This pressure determined by Mr. Brahtz analytically, may be readily found by graphics. Fig. 19(*c*) shows the graphic determination of the stress, σ_x, in the case of a long reservoir when one of the sides of the "angle of visibility" is practically horizontal. The stress in question is proportional to the area, $O N' M' M''$. The value of the radius, R, is arbitrary.

Value of the Poisson Ratio of a Dam Foundation and Displacement Formulas.—The author introduces the symbol, μ_2, as the Poisson ratio of the material comprising the dam foundation. One of the basic principles of soil mechanics is the incompressibility of soil particles. In other words, a soil particle is assumed to be capable of a change in its shape, but not its volume, in which case the value of Poisson's ratio would be 0.5. On the other hand, earth masses under action of their own weight produce lateral pressure, and the ratio of the lateral pressure to the vertical, or the "coefficient of pressure at rest", K, may equal 0.25 to 0.20, for example. If the elastic formula, $\kappa = \frac{\mu_2}{1 - \mu_2}$, is used for determining this coefficient of pressure at rest, the value of μ_2 would be one-fifth or one-sixth. Thus, a substance would have two different values of Poisson's ratio, which is obviously impossible. In the writer's opinion the conception of a "Poisson ratio" as applied to foundations, conveys a correct idea only in the case of elastic, dense rocks, such as some granites or basalts, and should not be used in other cases except, possibly, in the case of stiff clay foundations.

As to the displacement formulas, such as Equation (36) or Equation (40), in addition to containing an indefinite value of the Poisson ratio, μ_2, they are objectionable from the following points of view:

First.—They have been developed under the assumption that the earth mass is weightless and obeys Hooke's law unconditionally. An earth mass,

however, has been under the action of its own weight perhaps for millions of years; elastic deformations have long since occurred, and the construction of a dam involves not merely the application of a relatively light load for which Hooke's law holds, but a further considerable increase in stress which, at some depth, probably exceeds the elastic limit. At such high ranges of loading, deformations increase more rapidly than the corresponding loads, even in perfectly elastic bodies, and in the given case there is a danger of under-estimating the value of the displacement if Equation (36) or Equation (40) is used. Such an under-estimation may be termed a "positive error."

Second.—A displacement connotes a movement, or translation, of matter. In order to move even a molecule a certain minimum of energy is required. Hence, it is logical to assume that at a certain distance from the load, no displacement occurs which is contrary to the conditions of Equations (36) and (40). This fact may cause a "negative error."

Third.—It is known from every-day practice that elastic displacements in earth masses are accompanied by irreversible movements to such an extent that the elastic rebound after unloading a mass, sometimes reaches only 10% or 15% of the original displacement. No means has yet been devised for separating elastic, from inelastic, deformations; and only laboratory experiments furnish some bases for judgment as to the extent of the possible displacement. Positive and negative errors may compensate, and, in some cases, the observed settlement is found to agree closely with the value computed by elastic formulas; but by no means should this phenomenon be accepted as proof that such formulas would furnish satisfactory results in so far as displacements under any conceivable circumstances are concerned.

Shearing Stresses; Isochromatics.—Danger from shear is mostly a surface problem, and shear stresses are to be determined in the upper layers of the mass, close to its boundary surface. There is no danger of shear in deeper strata, where either compression or consolidation occurs. What matters in this connection is the maximum shearing stress, τ_{max}, as determined from the computed values of principal stresses or from isochromatics (see Fig. 12). The values of the shearing stress, $\tau_{x,\,y}$, acting in the horizontal plane (see heading "Application II.—Stresses in Foundations") are but of little interest for the designer of a foundation.

Attention should be called to Fig. 12(*a*) in which the shape of isochromatics reveals the overloading of the foundation of a dam near its edges. This diagram checks with photo-elastic experiments by other investigators when a punch is pressed against a two-dimensional elastic mass. It should be borne in mind, however, that loading experiments with rigid earth masses (such as sand) reveal overloading at the center of the base of the structure. This induces one to believe that isochromatics, as shown in Fig. 12(*a*), have a restricted meaning only and should be used when there is absolutely no doubt as to both elasticity and isotropy of the foundations, which evidently is not the general case. The same is true of Fig. 12(*b*).

Conclusion.—Applied mechanics, including the theory of elasticity, has been developed for the most part to find the answer to practical questions. It

has happened that the attention of the Engineering Profession is principally directed toward machines and framed structures; hence the unusual development of mechanics in the study of such subjects as beams, columns, shafts, plates, disks, etc. The study of "masses" or bodies possessing three great dimensions has been relatively neglected. The masses, in turn, may be subdivided into "unlimited" or semi-infinite and "limited" masses. In the study of stress distribution in semi-infinite masses there have been the works of such investigators as Bouissinesq, Michell, Carothers, and, in the past few years, of a number of research workers in soil mechanics. In the study of "limited" masses, such as concrete or masonry, notable progress has occurred lately, mostly due to research incidental to the design of dams, including photo-elasticity. In the latter category, the paper by Mr. Brahtz is a valuable contribution to the study of "limited" masses, and there is every reason to believe that his work along this line and his methods will be used to advantage not only by the builders of dams but also by workers in the allied fields of engineering science, such as soil mechanics.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE RELATION OF ANALYSIS TO STRUCTURAL DESIGN

Discussion

BY MESSRS. N. M. NEWMARK, AND L. E. GRINTER

N. M. NEWMARK,[22] JUN. AM. SOC. C. E. (by letter).[22a]—Perhaps this paper by Professor Cross will serve to divert attention from the elaboration of analytical technique to the development of a technique of design. Certainly the author is to be commended for raising the question of the proper interpretation of analyses and for pointing out a practical and useful philosophy of design. The classification of structural action presented in the paper is the first attempt of which the writer knows to establish a basis for a rational design procedure.

Whether or not the traditional process of structural design is practicable depends on how the forces and moments in a member are affected by revisions of the dimensions of that member and of other members in the structure. For example, a typical relation between strain (angular or linear) and size of member, for a particular member of a given structure under a constant condition of loading, is shown in Fig. 7. Curve (*a*) shows the relation for one set of given constant dimensions of all other members, and Curve (*b*) that for another set.

In Fig. 8 are shown curves for the relation between total force or moment in the same member under the same conditions as those for which the curves of Fig. 7 were drawn. The approximate ranges of "participation" and of "normal" action are indicated in the diagrams. Action in the intermediate range is characterized as "hybrid" action.

If, for practical variations in other members of the structure, Curves (*a*) and (*b*) are close to each other, the structure as a whole may be considered a "normal" structure; otherwise, it is a "hybrid" structure. The matter of efficiency or inefficiency depends upon whether or not the members in the

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in October, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1935, by Messrs. L. J. Mensch, and Russell C. Brinker; January, 1936, by Marshall G. Findley, Assoc. M. Am. Soc. C. E.; and March, 1936, by Messrs. I. K. Silverman, Bruce G. Johnston, and Harold E. Wessman.

[22] Research Asst. in Civ. Eng., Univ. of Illinois, Urbana, Ill.

[22a] Received by the Secretary February 19, 1936.

structure can act together at the desired working stresses. This generally depends on the over-all proportions of the structure as well as on the sizes of the individual members.

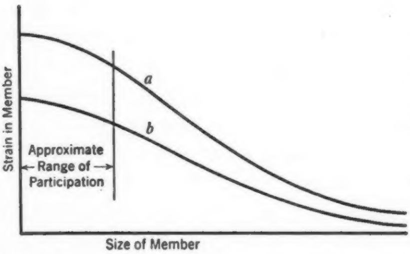


FIG. 7.—TYPICAL RELATION BETWEEN SIZE OF MEMBER AND STRAIN IN MEMBER FOR GIVEN CONDITION OF LOADING AND FOR TWO DIFFERENT SETS OF DIMENSIONS OF OTHER MEMBERS IN THE STRUCTURE.

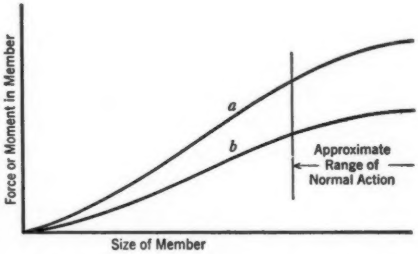


FIG. 8.—TYPICAL RELATION BETWEEN SIZE OF MEMBER AND FORCES ACTING ON MEMBER FOR GIVEN CONDITION OF LOADING AND FOR TWO DIFFERENT SETS OF DIMENSIONS OF OTHER MEMBERS IN THE STRUCTURE.

For a condition in which the structure is constrained to undergo a fixed deformation, a set of curves somewhat similar to those in Figs. 7 and 8 are obtained, but in general the "deformation" stresses are of a type corresponding to "participation" stresses. The fundamental difference between load stresses and deformation stresses may be stated as follows: For given applied deformations, the strains throughout the structure are independent of the absolute values of the modulus of elasticity; and for given applied loads, the forces in the individual members are independent of the absolute values of the modulus of elasticity.

Importance of Sources of Stress.—The relative importance of the various sources of stress must be considered in any attempt to revise the proportions of a structure or of its members. As an interesting example consider the case of a filled-spandrel arch bridge. Any effect of the spandrel walls may be ignored since this is purely an illustrative example. In general, dead load is by far the most important source of stress in such a structure. If the arch axis fits the pressure line for dead load, the dead load forces at any section of the barrel are practically independent of the variations in depth of the barrel. The structure is then a "normal" structure for dead loads; it remains "normal" even when the live load effect is considered, since the variation in depth to account for the live load stresses will be fairly small, and the effect of such variation on the maximum total stress will be practically negligible.

However, if such a structure were to be designed for live load only, it would very likely be "hybrid" since a variation in depth in one part of the barrel would effect the stresses in other sections to a fairly considerable extent.

Another question in this connection is worth raising. Consider the design of a continuous beam for relatively heavy concentrated live loads. If the designer limits himself to the choice of prismatic sections in each span, he may possibly arrive at a design in which the maximum fiber stresses in each span are equal. If, however, he attempts to choose proportions such that the

maximum stresses at all points in all spans are equal to the design stress, the problem of design, by any existing procedure, becomes almost hopelessly complicated.

Significance of Over-Stress.—The significance of over-stress in a structural member is a matter which has not received as much attention, perhaps, as it deserves. Certainly the literature contains few references to this topic. The author's classification seems to offer a basis for the study of the question. Professor Cross suggests that over-stress due to participation or deformation is not always dangerous. In some hybrid structures, also, over-stress loses its significance when the ultimate strength and nature of failure of the structure are considered, as in the case of the trussed beam discussed subsequently. Apparently, a great opportunity is available in this field for experimental research.

The classification offers a convenient method of estimating the possibility of the significance of chance factors in the properties of the materials, and in the manner of construction. In a "normal" structural type such factors have comparatively little influence. In a "hybrid" structure, they may be of considerable importance. Studies of the effect in an arch rib of chance variations of modulus of elasticity have been reported by Professor Cross.[23] The results seem to show that there is only a relatively small effect of chance factors on the stresses in an arch rib for the usual case in which the dead load is comparatively large. When the dead load is very small compared with the live load, chance variations might cause some concern. However, the effect of chance variations cannot be discussed independently of the significance of over-stress.

The Trussed Beam.—The problem of the trussed beam is of interest in throwing light upon some of the aspects of the use of the classification in design. Certainly, if the sag rods are quite small, the structure is "normal", with the beam "carrying the load", and with "participation" stress in the rods. Again, if the beam is very shallow and has an area of the same order as the sag rods, the structure is "normal", and acts as a truss with secondary or "participation" stresses in the beam. For intermediate cases, the structure is "hybrid."

For the case of a concentrated load at the center of the beam, the hybrid structure is "efficient" when the sag depth is approximately 1.5 to 2 times the beam depth (for the beam and sag rods both of the same material). For lesser sag depths, the sag rods will be understressed compared with the beam for all proportions of the structure. For greater sag depths the sag rods will be overstressed compared with the beam unless the beam is so small that the structure approaches a truss. When the beam is fairly small, it will be overstressed compared with the sag rods unless its depth is adjusted so as to reduce the flexural stresses. If the beam must carry loads at other points than at the center, the foregoing results are changed quantitatively but not qualitatively.

[23] "Dependability of the Theory of Concrete Arches", by Hardy Cross, *Bulletin No. 203*. Univ. of Illinois Eng. Experiment Station, 1930.

Of course, actually, the stresses in beam and truss may be adjusted either by putting initial tension in the sag rods or by slacking off on them, when the sag depth is less than, or greater than, the efficient sag depth, respectively.

Merely as an illustration, consider the case of a 30-ft beam carrying a concentrated load of 44 000 lb at its mid-point. It is desired to truss the beam to help support the load. Adjustments of the sag rods or stresses due to dead load are not considered. It is assumed in the following calculations that the vertical post is fairly large so that it does not deform appreciably under the load.

Problem 1.—The beam is an 18-in., 90-lb **I**-beam; and the sag depth is chosen arbitrarily as 36 in. from the center of the beam. What area of sag rod is required for working stresses of 18 000 lb per sq in.?

Discount about 2 000 lb per sq in. in the beam for direct stress. At a flexural stress of 16 000 lb per sq in., the beam will carry about 56% of the load. Designing the sag rods for the remaining stress, one finds that the area required is about 2.75 sq in. A review with this design indicates that the total stress in the beam is 17 600 lb per sq in., and in the rods, 18 200 lb per sq in. Evidently, an efficient sag depth was chosen.

Problem 2.—Under the same conditions as in Problem 1, the sag depth is taken as 54 in. A first estimate of the area required in the sag rod is 1.67 sq in.; but a review indicates a stress of 23 900 lb per sq in. in the rods, and a total stress of 14 800 lb per sq in. in the beam. Apparently, the sag depth chosen is inefficient.

By use of the traditional design procedure, the next trial design is an 18-in., 70-lb **I**-beam, and an area of 2.22 sq in. in the rods. This combination, if moments and forces are not changed, should give a stress of 18 000 lb per sq in. in the rods, and 20 100 lb per sq in. in the beam. However, a new analysis gives stresses of 21 900 and 15 900 lb per sq in. in rods and beam, respectively.

Repetitions of the traditional procedure lead very slowly to a balanced design. The preceding results suggest for a next trial a 54.7-lb **I**-beam and an area of 2.70 sq in. in the rods. Limiting the depth to 18 in. the designer finally ends with a 48.2-lb beam, and rods of an area of 3.0 sq in., for which the stresses are calculated to be 18 600 and 16 200 lb per sq in. in rods and beam, respectively.

If the problem had been to reinforce the original beam, the designer would have found eventually that approximately the same rod area would have been required as in Problem 1, namely 2.7 sq in. The beam, however, would have been working at a stress of 12 300 lb per sq in., compared with 17 300 lb per sq in. in the rods.

In this problem the question may well be raised as to the significance of over-stress in the sag rods when the beam is understressed. If the stress in the rods reaches the yield point the beam will begin to take a larger share of the load. The structure might adjust itself in a manner analogous to slacking off the tension in the sag rods by increasing their length slightly. The writer does not believe that it is well to depend on such stress relief in general, but the question does deserve some consideration.

Problem 3.—With other conditions the same as in Problem 1, consider a 24-in., 79.9-lb **I**-beam, and a sag depth of 36 in.

In the same manner as in Problems 1 and 2 a first estimate of the area required in the sag rods is 1.57 sq in. The calculated stresses, however, are 18 800 lb per sq in. for the beam and 16 000 lb per sq in. for the rods. If the rods are reduced in area and the beam is increased in size, as is indicated by the traditional procedure, the ultimate effect will be to eliminate the rods entirely, and the entire load will be carried by a larger beam.

If it is a matter of reinforcing the given beam, the designer will have to resign himself to the use of a lower working stress in the rods than in the beam. With an area of 1.90 sq in. in the rods, the stresses will be 15 000 and 18 200 lb per sq in. in rods and beam, respectively.

One could have anticipated the final designs obtained in these three problems, however. It is not difficult to estimate what simultaneous stresses are required for continuity with given dimensions of sag rod and beam.

Arches Continuous with Their Superstructure.—Certain limiting cases of the arch with integral spandrel structure are "normal", but practically all such structures of the proportions usually built are hybrid.

An arch with a comparatively heavy rib and flexible columns and a deck supporting relatively light live loads is a normal structure with participation stress in the deck and columns added to the normal stress in these members due to their action in the continuous deck frame.

Another normal type is the structure consisting of a flexible rib with a stiffening girder. There will be flexural participation stress in the rib, in addition to the normal stress due to the direct thrust in the rib.

A third type of structure having "normal" characteristics in certain cases is what might be termed an arched Vierendeel truss. This structure must have quite rigid columns. The corresponding simply supported Vierendeel truss would be normal, but fixing the supports introduces some complications. In this connection, it is important to note that the fixed arch rib has hybrid characteristics for live loads, as was pointed out previously.

Other normal arch structures may be secured in various ways by the manner of construction or design involving the use of articulations and flexible sections. Such structures have well marked participation stresses at such articulations and pseudo-hinges. Of course, not all structures with hinges and points are "normal."

In special cases applying to certain other arch structures the controlling relations are comparatively clear, as Professor Cross has indicated. The flitch-beam concept applying to the structure with flexible columns has considerable value. Use was made of this concept by Rankine[24] and Considére.[25] An analysis preliminary to design may be developed along the lines suggested by Professor Cross' treatment as follows: The deck girder will have primary stresses due to its action as a continuous beam. These stresses are due to the dead load of the deck only, and the entire live load. The arch rib will

[24] See "Civil Engineering", Seventh Edition, 1871, pp. 313–314; and other editions at approximately the same page numbers.

[25] See, for example, "Cours de Beton Armé", by A. Mesnager, Paris, 1921, p. 198.

take all the thrust, most of which is due to dead load. Then the difference between the working stress and the primary girder stress is available to resist interaction flexure in the girder. The difference between the working stress and the stress due to direct thrust in the arch rib is available to resist flexure in the rib. The stress due to interaction flexure in the girder is to the flexural stress in the rib approximately as the depth of the girder is to the depth of the rib. The rib and deck resist the moment, except for primary moments, approximately in proportion to their moment of inertia.

A few rough figures will enable an estimate to be made as to whether a tentative combination of depths of girder and rib is efficient or inefficient, and will give some idea as to what can be done about it. For example, if the girder is found to be overstressed, it must be remembered that increasing the depth of the girder may increase the interaction stress although it decreases the primary stress.

For the general case in which the columns must be of intermediate stiffness there seems to be no convenient method of estimating in advance whether the structure can act as it is assumed to act in a preliminary analysis. Use of the traditional procedure of successive analysis (usually by means of a model) and design does not appear to be very promising. Such a procedure might easily be misleading, particularly where a re-design is based on an analysis which is not repeated for the new design. It is open to question whether a better structure is obtained by such a procedure than by neglecting interaction entirely.

Other Problems.—The fundamental philosophy presented by the author has wide applicability. Some of its possible uses have been mentioned, but there are many other cases in which it is useful. As an illustration consider the problem of the stresses in a bearing-plate supporting a concentrated load on an elastic solid.

If the plate is very flexible compared with the solid, the latter deforms in the same manner as under a load directly applied. The stresses in the solid are of the nature of "normal" stresses. The plate, in conforming to the configuration of the solid, has stresses of the nature of participation stresses. If the plate is very stiff, the pressure between it and the solid approaches a certain limiting distribution, and the stresses are "normal" in both plate and solid. If the bearing-plate is neither very flexible nor very stiff, the pressure distribution under it varies considerably with any change in thickness, and the stresses are certainly "hybrid."

The classification has applicability to many other problems concerned with what is sometimes called "internal stress", as well as to problems concerning so-called "frame" structures.

Concluding Remarks.—The profession is indebted to the author for his presentation of a practical philosophy of design. It is to be hoped that his paper will awaken interest in methods of design. Perhaps in the future more energy will be devoted to securing quantitative data for the design of structures rather than to the development of variations in existing procedures for their analysis.

L. E. GRINTER,[26] ASSOC. M. AM. SOC. C. E. (by letter).[26a]—To his important contributions on the theoretical analysis of continuous frames, Professor Cross has added an enlightening paper concerning the relation of analysis to structural design. His subdivision of all structures into normal and hybrid types cuts ruthlessly through the field with little regard for the classical arrangement into statically determinate and statically indeterminate groups. This point of view is at least refreshing, and it seems to be considerably more than that.

Statically Determinate Structures.—The writer has often wondered whether he has ever actually seen a practical structure that could be properly classified as statically determinate. Simple beams are decidedly unusual. What is commonly termed a simple beam (that is, a steel floor joist or roof joist) is frequently encased in concrete poured continuously with other beams through the connection of a reinforced concrete floor-slab or roof-slab. Thus, this structure, which is analyzed as statically determinate, may be one of the most complex of all statically indeterminate constructions—a continuous flat slab with partly discontinuous stiffening ribs supported in part by the columns and in part by restrained edge beams. A search for a statically determinate cantilever structure will not lead to much more satisfactory results since in common with three-hinged arches they become not only statically indeterminate, but mathematically indeterminate when corrosion produces an unknown pin friction. They, too, are usually constructed so that there is partial continuity through a floor system.

This comparison of structural types could be carried on indefinitely. Surely the lower chord of a pin-connected through truss ought to be a statically determinate member; but, instead, the writer recalls making a railway bridge test for live loading for which numerous strain-gauge measurements on the eye-bars of a lower chord member showed an average stress of little more than one-half the theoretical value. One could explain this phenomenon in part by the self-evident fact that the floor system was functioning as a part of the lower chord, but this general observation would be of little value in an attempt to analyze or to design a similar structure. The fact is that this simple span truss was acting in a most complex manner, that it was internally statically indeterminate, and that the manner of fabrication and erection had a most important effect upon its stresses.

From these and other observations, one might conclude that the classical distinction between statically determinate and statically indeterminate structures has its main justification in the classroom where its value is largely a matter of convenience to the student and the teacher. For the practical designer this distinction is dangerously convenient and, therefore, of questionable importance. Hence, the subdivision suggested by the author is at least worthy of consideration.

Impressed Distortions and Induced Distortions.—Professor Cross has made a division between what he terms deformation stresses and participation

[26] Prof. of Structural Eng., Agri. and Mech. Coll. of Texas, College Station, Tex.

[26a] Received by the Secretary February 24, 1936.

stresses. A comparable physical distinction that the writer has been in the habit of making depends upon whether the corresponding distortion is impressed upon the structure or induced in the structure by its own action. Evidently, temperature effects are impressed upon the structure from an outside source. Similarly, the tower of a suspension bridge may have a bending deflection impressed upon it because of a reduced sag of the cable in the side spans when the center span is loaded. At least, this action is external to the tower itself and may thus be classified as an impressed distortion. On the other hand, ordinary secondary stresses in truss members are caused by joint rotation in the plane of the truss which is an induced distortion. Apparently, however, the division line is less clear when one considers such effects as rib-shortening in arches. This action is quite clearly the same as temperature effect or spread, both of which are produced by impressed distortions. Nevertheless, rib-shortening stresses are secondary flexural stresses produced by the shortening of the rib from primary compression, and thus should be designated as "induced stresses." Hence, one observes that impressed and induced distortions merge one into the other.

Professor Cross has attempted to clarify the aforementioned difficulty by observing that time flow will eliminate or reduce a deformation stress (the consequence of an impressed distortion), but that a participation stress (the consequence of an induced distortion) will be unaffected by such plastic flow. In general, this distinction holds true although it may be desirable to add that the distribution of all stresses can be affected by such plastic flow. These observations explain the writer's preference for the terms, "induced stress" and "induced distortion", as more descriptive of the physical picture than the terms, "participation stress" and "participation strain." Certainly these secondary stresses participate very slightly in the job of supporting the loads; instead, they are induced by primary strains much to the designer's disquietude.

Two statements of the author will be repeated for study: (1) "Thus it is generally known that increasing the moment of inertia without changing the depth of truss members will affect the secondary stresses only as the primary stresses are affected"; and (2), "the designer finds it convenient to predetermine such strains [impressed distortion], reduce them to equivalent stresses, and deduct these stresses from the working stresses available for load-carrying capacity." Although the writer thinks that Professor Cross has been too generous in concluding that the information referred to in Statement (1) is generally known, he certainly agrees to its importance. Statement (2), although simple in itself, must be complicated by the fact that only the total distortion is evident and that the unit strain is dependent upon the shape of the axis of the member and upon the variation of its cross-section. Hence, for the general case one must admit that even final deformation stresses (impressed distortions) cannot be found until the design is completed. This is just another of those cases in which the designer cannot "foresee the action" and falls back upon a cut-and-try procedure.

Visual Structural Action.—The writer would mention another classification of structural action which in many points agrees with the subdivision

suggested by Professor Cross. However, a few structures will be found to have been shifted from the complex to the simpler classification, or *vice versa*. A comparison of the two points of view seems particularly useful since each presents factors for consideration that the other misses.

The subdivision to be suggested is made dependent upon a criterion which can be stated forcefully in the words of Professor Cross, "that the designer either does or does not know what he is doing." In the writer's words this becomes a subdivision into "visual structural action" and "non-visual or concealed structural action." As another criterion that will be of some use, it may be stated that this classification of visual structural action will include all academic statically determinate types and, in general, those indeterminate types that involve, at least theoretically, no more than three redundants. In some few cases a structure having more than three redundants may possibly be arranged to fit into the simple classification of visual structural action. Examples will be given.

The limitation of three redundants is suggested because it is quite evident to the instructor that the limit of visual study commonly is reached when three interacting influences exist. The interaction of two redundants is relatively simple but the number of possible interwoven patterns is about tripled with three redundants. A visual study of the interplay of four influences is not ordinarily within the grasp of other than a highly trained mind. As an example of the usefulness of this classification, one notes that the problem of two crossing beams (deflected equally) which were placed in the complex subdivision by Professor Cross fall into the simple subdivision in this discussion. The analysis and design of such beams, although not feasible by the "guess and revise procedure", is possible in a direct manner, as suggested by the author.

Non-Visual or Concealed Structural Action.—In this classification is found those structures and parts of structures for which an adequate design procedure does not now exist. It is no great credit to the profession, however, that it includes so many ordinary structural types. Hence, it is worth calling attention to the fact that in this subdivision there is included many concrete arches, most tall building frames, flat slabs, continuous footings, bridge abutments built continuously with their wing-walls, arched dams, and even many of the so-called statically determinate structures because of the partial continuity introduced by common methods of construction. In other words, with few exceptions, an adequate design procedure to-day is available only for those structures in which the interaction of the structural parts can actually be visualized. The designer will do well to select and construct structures within this classification until direct procedures for the design of complex structures are made available. When he cannot do this, he should "go slow" in making his design.

Parallel Beams.—Two cases of parallel beams seem worthy of investigation. In wind-stress studies, parallel beams of different depths and perhaps even of different lengths may be forced to rotate through equal end angles. Then,

since θ varies as $\dfrac{M\,L}{E\,I}$ and $M_s = \dfrac{S\,I}{c}$, it is evident that for equal θ-values the relative stresses in the two beams are defined as follows:

$$\frac{S_1}{S_2} = \frac{\dfrac{L_2}{c_2}}{\dfrac{L_1}{c_1}} = \frac{\dfrac{d_1}{L_1}}{\dfrac{d_2}{L_2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

in which $\dfrac{d}{L}$ is the depth-length ratio of the beam. Evidently, the only geometrical shape that will permit such beams to be stressed equally is that of equal $\dfrac{d}{L}$ - values.

When the deflections rather than the end slopes are identical for the two beams, the deflection varies as $\dfrac{M\,L^2}{E\,I}$ and the relative stresses in the two beams are defined as follows:

$$\frac{S_1}{S_2} = \frac{\dfrac{L^2_2}{c_2}}{\dfrac{L^2_1}{c_1}} = \frac{\dfrac{d_1}{L^2_1}}{\dfrac{d_2}{L^2_2}} \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Hence, these beams must have equal $\dfrac{d}{L^2}$ - values in order to be stressed equally. This would be the writer's definition of efficient design. In any case such beams furnish an illustration of a statically indeterminate structure properly classified as hybrid under the author's subdivision, but as visual structural action by the writer.

The Vierendeel Truss with Parallel Chords.—This is another structure properly classified as hybrid by Professor Cross, but still easily analyzed and readily designed because of the visual characteristics of its structural action despite the fact that it is highly indeterminate. A few common sense basic assumptions can be made for the purpose of a preliminary analysis and design by statics. Then, in the usual case, an analysis will show that the first design was satisfactory. The few necessary revisions are easily determined because of the visual characteristics of its structural action.

The distinction between the Vierendeel truss and a building frame is that the truss is essentially a regular structure, whereas, the building frame is in most cases irregular. Commonly, there are no reasonable assumptions that can be made to lead to a preliminary design of a highly irregular windframe that will not necessitate serious revision of sections subject to the difficulties inherent in the revision of any structure the action of which cannot be visualized. Of course, the Vierendeel truss can be hopelessly complicated by the use of a floor system such that the resultant structural action will be beyond visualization. A curved top chord will introduce additional complications.

Rectangular Wind Frames.—The author proposes an interesting design procedure for a regular wind frame. His discussion apparently is intended to apply to a building of medium height since it is assumed that the column cross-sections will be unchanged by the effect of wind. If such a building frame has been designed by a conventional method, it is likely that overstresses will be found when an analysis is performed by a more exact method. However, one need not run headlong into the difficulty pointed out by Professor Cross. Naturally, an increase in size (depth and moment of inertia) increases the stiffness of the overstressed girder, causes it to resist more wind moment, and increases its fiber stress still more. One frequently finds it necessary to reduce its depth, thus reducing the fiber stress for a given moment. At the same time, it may be desirable to stiffen adjacent girders to relieve the overstressed girder of its excess moment.

Of course such a procedure is difficult to apply to a highly irregular structure, but there is no particular reason why the design of a structure as complex as an irregular tall building frame should not consume considerable time. The author's proposal that the design should be based upon assumed points of contraflexure, assumed shear distribution, and design stresses proportional to the depth-length ratios of the girders (equal end slopes) would be practicable for a very regular frame. However, the important need is for a design method that will make the design of an irregular building frame less tedious.

Conclusion.—The writer has found in this paper much material that will require slow digestion. The inference is clear that direct design is the ideal toward which one should strive. The cut-and-try or guess-and-check procedure fails in many cases unless applied with more than ordinary intelligence. Finally, hybrid structures, in which the type of structural action is concealed, in which "self interference" exists, or in which one attempts to "put secondary stresses to work", offers little promise of economy, but far more promise of confusion and poor design.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

INFLUENCE OF DIVERSION ON THE MISSISSIPPI AND ATCHAFALAYA RIVERS

Discussion

BY LEO M. ODOM, ASSOC. M. AM. SOC. C. E.

LEO M. ODOM,[15] ASSOC. M. AM. SOC. C. E. (by letter).[15a]—Certain proofs and deductions are set forth in this paper, based on data and theory, which merit the consideration of students of the subject of the control of the Mississippi River and its tributaries. Among such students may be counted, to a greater or less extent, practically every engineer who resides in the Mississippi Valley.

Apparently, the author begins with the hypotheses: (1) That at points on the Mississippi River not affected by diversion operative at all stages, the same volume of water is now passing at the same elevation as in earlier years; (2) that the Mississippi River has conformed itself to "the" hydraulic theory and increased its slope as the volume has been diminished by diversion; and (3) that the Atchafalaya River has conformed itself to "the" hydraulic theory and has flattened its slope as its volume has been increased through diversion. The hydraulic theory from which the second and third premises were deduced is presented with discussion to show how it was interpreted to arrive at the hypotheses and to explain the causes underlying the action. Selected data from the records of the Mississippi River Commission for the years 1882 to 1932, inclusive, are presented and discussed. By consideration and interpretation of the data, the author concludes: (*a*) That the hypotheses are correct; (*b*) that the first is correct through simple consideration of the data presented; (*c*) that the data presented show that the Mississippi River has actually increased its slope and that the Atchafalaya River has actually decreased its slope during the period in which records were examined, as set forth in the second and third premises; and (*d*) from the fact that the first premise is true, and from consideration of the theory cited, that the cause of the noted phenomena in connection with the second and third premises is the fact that diversion is permitted from the Mississippi River into the Atchafalaya River at practically all stages.

NOTE.—The paper by E. F. Salisbury, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[15] Res. Engr., Corps of Engrs., U. S. A., 1st New Orleans Dist., New Orleans, La.

[15a] Received by the Secretary February 24, 1936.

Besides concluding that his hypotheses are proved, the author arrives at certain other conclusions and hypotheses. He deduces from data presented as to the Pointe a la Hache crevasse and study of the discharge measurements at Arkansas City, Ark., near which many crevasses have occurred, that overbank diversion does not cause the same action as "year-round diversion" and that if overbank diversion were substituted for the present continuous diversion through Old River, the main channel below that point would be practically restored to its original discharge capacity. He theorizes that the difference in effect between year-round diversion and the overbank diversion is due to the fact that (1) the greater portion of the silt load that is handled along the bed of the channel is accorded less disturbance with overbank diversion only; and (2) with diversion occurring only at stages above bank-full, an agency exists, acting for 365 days per yr, to clean out any deposition of silt left after the short duration of overbank diversion, which agency presumably does not exist in full force at points of year-round diversion.

He concludes from the fact that the Mississippi River had not previously taken the shorter course to the sea offered by the Atchafalaya, that the normal action of the stream, determined by its silt load, caused the longer route to be followed and the shorter to be blocked. He deduces from comparative data that the total discharge of both rivers at equivalent gauge readings is less now than formerly, which he explains to be the result of the action set forth in his hypotheses.

The writer concludes from the remarks in the content of the paper that the first premise as stated in the "Synopsis" should also include cut-offs as well as the diversion referred to, as affecting the author's choice of gauge stations for analysis in this regard. Except for this fact, and the warning that the premise can hardly be supposed to be proved by consideration of so few stations in the total length of the river, no more will be said about this premise.

As to the last two premises, proof will be offered in this discussion that "the" hydraulic theory referred to therein is not at present generally accepted and that evidence is against its application to the Mississippi River. The possibility of other interpretation of the data presented will be advanced, and the probability of causes other than the fact of diversion for the bar below Red River Landing will be discussed.

The hydraulic theory referred to in the second and third premises is apparently the theorem of Guglielmini[2] cited under the heading, "Introduction." Commenting on this theorem, Humphreys and Abbot did not state as a fact the quotation attributed to them in the paper[15b]. The entire discussion in the "Report on Physics and Hydraulics of the Mississippi River", which contains the statement of Guglielmini's theorem and the aforementioned quotation is stated therein at the beginning of the paragraph to be an "extract from the writings of Maj. J. G. Barnard, Corps of Engineers, U. S. Army", and is given for the express purpose of refutation.

[2] "Report on the Physics and Hydraulics of the Mississippi River", by A. A. Humphreys and H. L. Abbot, 1861 (reprinted 1876), *Professional Papers No. 4*, Corps of Topographical Engrs., U. S. A., p. 415.

[15b] The first principle of this theorem is misquoted and should read: "(1) The greater the quantity of water a stream carries, the less will be its fall."

Commenting on this theorem and Major Barnard's remarks, Humphreys and Abbot state: "It will be noticed that two important assumptions are necessary to support this reasoning: First, that the bottom of the Mississippi River is composed of its own alluvion, which can be readily acted upon by the current; and, second, that its water is always charged to the maximum capacity allowed by its velocity."

The first assumption is then attacked on the grounds that the bed of the river is not composed of its own alluvion, but of a tough and stiff clay which they suppose to have had its origin in a more ancient geologic period than that of the river itself. This supposition as to the ancient geologic formation of this clay probably is incorrect, but it is still a fact that there is a very extensive clay stratum interspersed with shell and sand pockets throughout the entire valley from the ancient delta to the sea. This clay was more probably formed by the deposition at its mouth of the fines and colloids in the sediment carried by the river as it came into contact with the salt water of the ancient bay. Regardless of its origin this clay exists; it is quite compact and is not readily scoured, and forms the bed of both the Atchafalaya and the Mississippi Rivers. The existence of this difficultly erodible clay has been the cause of the failure of the Atchafalaya to cut itself an hydraulically efficient channel to the Gulf and has resulted in the spreading of that stream, past the confining influence of its levees, into numerous shallow branches (which in times of flood overflow over the entire basin), the frictional resistance of which system retards the velocity in the channel and prevents its discharge from increasing in the same ratio as if it, too, had a single channel to the sea.

The second assumption that Humphreys and Abbot state is necessary for the river action to conform to the hydraulic theory—that its water is always charged to the maximum capacity allowed by its velocity—is proved by them to be wrong by presentation of the analyses of sediment content of the river water at various velocities, which analyses were conducted in 1851 and 1852 and 1858 at Carrollton, La., and Columbus, Ky. They state that, "at the date of highest water the river held in suspension little more sediment than at dead low water." They state[16], also, referring to tests made in 1851: "When the quantity of earthy matter held in suspension was greatest, the velocity was 2 feet per second less than the greatest velocity."

In a paper on "Permissible Canal Velocities" by the late Samuel Fortier, M. Am. Soc. C. E., and Fred C. Scobey, M. Am. Soc. C. E., which constitutes a final report of the Special Committee of the Society on Irrigation Hydraulics, the following statement[17] is made:

> "It is believed that there is a broad belt of velocities between the two 'critical' velocities, within which silt already loosened or brought in through a head-gate will remain in suspension while the bed nevertheless will remain undisturbed as regards scour. It is easy to show the absurdity of accepting the laws of silting as giving immediate answer to the laws of scour."

[16] "Physics and Hydraulics of the Mississippi River", p. 674.
[17] *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 941.

Any number of references and detailed sediment analyses disproving the theorem of Guglielmini could be cited from published documents of the U. S. Corps of Engineers, but it is believed that, with the complete citation of one of the author's own authorities and the foregoing additional reference, the writer has shown that part of the progress of the science of river hydraulics since the Seventeenth Century has consisted in the abandonment of the erroneous theorem.

The data presented are found to have been accurately copied from the published documents of the Mississippi River Commission. The choice of bank-full stage as the proving ground for the hypotheses and the "efficiency" assumption will be accepted although bank-full stage occurs too seldom in the 365 days to be sure that it is as logical a stage on which to base the proof as a lower one.

If the theory by which the author arrived at his hypotheses had been one generally accepted by the Engineering Profession and data had been presented for down-stream points which bore some relation to the data presented for Red River Landing, the writer would have been more inclined to have accepted the second hypotheses as borne out by the data. Without adequate theoretical basis, however, the statement that the Mississippi River has increased its slope as the volume has been diminished by diversion, rests entirely upon the gauge records at Red River Landing. At Carrollton—the only down-river point treated—the gauge height for bank-full discharge has not varied appreciably in the last 45 yr. As one outstanding example of a case wherein the data do not even show a continuous growth of bar at this point, the interpolated gauge height given for the chosen discharge at Red River Landing was 49.4 in 1903, whereas, in 1932, it was 49.0. The entire period of study was less than twice this interval. That a bar has formed and at times washed out below Red River Landing is evident and known to every one familiar with the river. The formation of this bar might as well be due to the angle of diversion or to the bend in the river, or both, as to the mere fact of diversion.

There are three points in the lower river where diversion occurs at all stages: Baptiste Collette's Bayou, 83 miles below New Orleans in the east bank; Cubit's Gap, 91 miles below New Orleans, in the east bank; and The Jump, 84 miles below New Orleans in the west bank. No records are kept on these streams ordinarily. Measurements during the flood of 1927 show a maximum discharge through Baptiste Collette of 13 689 sec-ft, through Cubit's Gap, of 134 288 sec-ft, and through The Jump, of 32 511 sec-ft. No shoaling of the main stream in the vicinity of these outlets has ever been noted.

At Head of Passes, 94½ miles below New Orleans, the river breaks up into three principal outlets: Pass a l'Outre, South Pass, and Southwest Pass. The average discharge through the passes in percentage of the total volume at Head of Passes is approximately: Pass a l'Outre, 42.5%; South Pass, 17.5% and Southwest Pass, 42 per cent. A point bar forms at the head of Southwest Pass and a tendency to scour is noted at the heads of the other two passes. Sills were placed across the head of Pass a l'Outre in 1876 and 1900 in an attempt to increase the flow through the other passes. The record of

the action has been that the discharge through Pass a l'Outre was not reduced, but a turbulence was created which scoured the bottom of the channel to a depth of 122 ft between Pass a l'Outre and South Pass. A mat sill placed across the Head of South Pass in 1917 for the purpose of holding stationary the discharge in South Pass which had increased from 7% of the river's flow in 1891 to 14.7% in 1917, did not succeed in its purpose, but produced a hydraulic jump that scoured the bottom of the channel of the pass below the sill to a depth of 129 ft. The sill dams mentioned were similar in construction to that referred to by the author.

It is seen from the foregoing that the fact of diversion does not necessarily create bars in the main stream. The point bar which forms across the head of Southwest Pass is due to the direction of the thread of the current across Head of Passes and the fact that Southwest Pass takes off on a curve from the course of the river above this point.

It is also apparent that sill dams have not been found to be efficacious in the control of discharge. The percentage of discharge has been found to be regulated by the total resistance to flow offered by a channel and regulatory works offering resistance relatively minor in comparison to this total have little or no effect.

As to the third of the three premises, practically no proof is offered. Much is made of the fact that the average cross-sectional area of the stream increased 47% during the period embraced by the study. It is evident that practically all this large increase in average cross-sectional area is caused by the enlargement in the lower end due to extension of levees. At Reach 6, for instance, the enlargement has been only 4.17 per cent. The author is correct in stating that the capacity of the Atchafalaya has not been increased by its increased cross-section since this increased cross-section has not extended to a point where the stream can finally discharge freely. Increased cross-section will be of advantage only when and if levees are extended to Grand Lake and a pilot cut furnished between them, as the Atchafalaya probably never will be able to cut itself an efficient channel through the stiff clay of the basin. Its enlargement at its source, due to scour, when it acts as the outlet for the relatively clear back-water area of the upper basin, is also useless as far as getting any water out the other end is concerned.

The author concludes that the fact that the Mississippi has never taken the shorter course to the sea offered by the Atchafalaya proves that the longer route is the one determined by Nature in its handling of the silt load. Another hypothesis is offered herein taking into account observed action at the Passes, a study of the course of the Mississippi River below Red River Landing, and the conclusions of geologists.

Geologists state that at one time the Mississippi River discharged into an estuary which reached farther north than Red River Landing and that in time it has completely filled this estuary and extended its delta out into the Gulf. The material of which this fill is formed, to approximately mean Gulf level, is preponderantly a stiff silty clay interspersed with sand and shell pockets with apparent remains of ancient forests found at different levels below mean low Gulf level. The toughness of this clay is attributable to the

flocculation of the colloids when the silt-laden waters came in contact with the Gulf. The remains of forests give rise to the assumption that the old fill has settled and compacted and that new fill has formed on top of it in a manner similar to that occurring at the mouths of the river to-day.

As the river built itself out of its estuary, it became more and more acted upon by the littoral currents and littoral drift and the prevailing winds. The effect of these agencies is quite apparent at the mouth of South Pass. There, the sea channel at the entrance makes an angle of 36° with the axis of the Pass, and the sediment of the river is piled up on the west bank of the channel. The river does not flow into this sea channel, which is kept clear by the littoral current, but spreads out in a fan-shaped area on top of the salt water. When the bar becomes so extensive to the west that the friction is greater than the force it would take to move the salt water out of the channel, the river water will penetrate farther into the channel and thus its bed is continued on out to sea.

From Baton Rouge, which is near the limit of the ancient estuary, to the Gulf, the easterly trend is particularly to be noted, as past this point the mouth of the river was subject to the full action of the winds and Gulf agencies. From Red River Landing to Baton Rouge it will be noted that the river is near the eastern limit of the estuary with the main delta to the west.

It is believed that this hypothesis offers a better explanation for the present location of the main river than that based on the theorem of Guglielmini offered by the author; especially since it is evident that it has not always been 310 miles from Red River Landing to the Gulf by way of the Mississippi River.

The land structure now existing between the head of the Atchafalaya River and the Gulf creates an entirely different condition than that which existed when the mouth of the Mississippi was in the vicinity of Red River Landing. The sea agencies and prevailing winds which closed off the westerly outlets at that time would now affect only the mouth of the Atchafalaya.

As to the conclusion of the author that overbank diversion offers a solution to the evils attributed to continuous diversion, it is remarkable that he should have chosen as his basis for argument against continuous diversion the same theory advanced generations ago against overbank diversion.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABLE CHANNELS IN ERODIBLE MATERIAL

Discussion

BY MESSRS. E. S. LINDLEY, J. C. STEVENS, C. R. PETTIS, HARRY F. BLANEY, AND SIGURD ELIASSEN

E. S. LINDLEY,[5] M. AM. SOC. C. E. (by letter).[5a]—Throughout the paper, in the text, and on the diagrams, Kennedy's data are ascribed to the Lower Bari Doab Canal, whereas they were observed on the Bari Doab Canal, which became "Upper" on the construction of another canal many years later. The latter takes off from the same river about 160 miles lower in its course; it was originally designed to Kennedy velocities, but experience has shown that (writing from memory) about 0.80 of these would have been better.

The Garrett diagrams do not offer any additional silt theory, but are only an alternative tool for design with provision for following the Kennedy theory. They originated in the Central Provinces of India, before diversions of rivers for irrigation had been begun there.

As to the writer making "no suggestion that these relations might be influenced by the quantity or quality of silt", there was no need even to "suggest", in 1918, what was so generally known and accepted in Indian irrigation practice. The designing diagram in the writer's paper[6] provides for varying the relation of velocity both as to depth and to width, for local conditions.

Although it has since been superseded by Lacey's later work, the writer would like to refer to a development of his work quoted in the paper; this was published in the *Panjab Irrigation Technical Review* in 1925, and its diagrams (Plates 7 and 8) with a brief explanation, are given also in *Panjab Irrigation Nomograms.*[7] These diagrams increase the widths for very large channels in accordance with data obtained, as the former paper was practically complete. Whereas the former paper dealt mainly with the data examined, the latter considered more the points to be kept in mind in designing.

NOTE.—The paper by E. W. Lane, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1936, by R. C. Johnson, M. Am. Soc. C. E.

[5] Superintending Engr., Indian P. W. D., Panjab Irrig. Branch, (*Retired*), Wotton-under-Edge, Gloucester, England.

[5a] Received by the Secretary January 8, 1936.

[6] "Regime Channels", by E. S. Lindley, *Proceedings*, Punjab Eng. Congress, 1919, Plate III.

[7] On file for reference in Engineering Societies Library, New York, N. Y.

A fundamental point, now widely accepted in India, but not brought out by Professor Lane, is that a given volume of flowing water carrying a given proportion of a given quality of silt tends to form a channel of which the depth, width, and gradient are all fixed by the said volume and silt. In the writer's papers, attention is called to a number of factors that are likely to operate against this tendency, temporarily, but more powerfully.

This problem is too complicated for rational solution; and it is unlikely that the rational approach will yield more than an occasional "pointer" in the right direction. For example, it may confirm or may contradict Lacey's suggestion that velocity should be correlated with hydraulic mean radius instead of with depth. For effective advance by empirical methods it is almost essential to have a "yardstick" by which to test all obtainable data, and to use these data to improve the yardstick periodically. In India and in the irrigating countries in touch with it, such a rule was provided by the Kennedy theory, followed by the others mentioned in the author's paper.

To allow for the considerable volume of data needed to yield any real information, the labor of calculating factors from observed data, and one's weak and sinful human nature, it is further almost essential to put this yardstick into the form of diagrams to facilitate the examination of data and the design of new channels which can be watched. It is highly probable that Kennedy's theory would have been still-born, if he had not also provided diagrams which facilitated work that had to be done in any case.

If the paper conveys the impression that it is now anywhere possible to design a channel precisely to the dimensions which Nature will accept for local conditions, that impression is false. Where "design" is applied to an established channel, and consists of regularizing and accepting actual conditions, this will be accurate as long as the silt-draw of all intakes above, and of all offtakes, does not change materially. Where a new channel is being designed without data for the dimensions demanded by the silt, a value must be judged for the silt factor in the formula in use, and design must err on the side on which error causes the least inconvenience when Nature gives its decision. With designing diagrams available for the different formulas, it is not difficult to calculate a number of channels indicating the range of probability if the silt factors have been wisely chosen. For the choice of silt factors, the only help to experience and judgment is in a suggestion to equate Manning's flow formula and the silt formula, and insert values of observable data taken from the river. This suggestion is due to Mr. F. R. Burkitt, of the Panjab Irrigation Department; it has not been published by him, but is developed for Kennedy's silt formula in *Panjab Irrigation Nomograms* (Plate 6*a*).

The enormous silt charge described in the paper, and also in a paper[8] by the late Carl Ewald Grunsky, Past-President, Am. Soc. C. E., entitled "The Head-Works of the Imperial Canal", is striking: even if all channels are designed to pass it in suspension, it must cause difficulty when it reaches the irrigation ditches and the fields, and so must be reduced to reasonable quantities. This should be feasible if advantage is taken of the experience

[8] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 262.

available in places at which considerable advance has been made in designing offtakes to exclude silt. When that has been done, it will soon be clear whether the channels depart so materially from the dimensions of those studied.

J. C. STEVENS,[9] M. AM. SOC. C. E. (by letter).[9a]—The anomalous behaviors noted between Egyptian, Indian, and American canals and rivers in alluvium are probably due to fundamental differences in the character of the alluvium. The author discounts the effect of colloids and thinks their apparent effect may be attributed to the silt load of the stream.

When segregated, colloids appear jelly-like, whereas the cementing effect attributed to them may well be given by coarser material, such as fine clay, that would not fall in the colloid group at all. It is a common observation that clay deposits that have had an opportunity to consolidate become highly resistant to erosion as long as the beds remain intact. Once the particles become loosened from the bed by exposure and weathering, however, they are readily moved by very low velocities. Many such examples of this fact may be cited in clear streams and canals of the West.

If the silt carried by a canal contains a considerable quantity of fine clays or colloids it begins to consolidate as soon as it is deposited, and once deposited it requires much higher velocities to dislodge it again than to keep it moving. The writer believes, therefore, that the colloids and fine clays in the alluvium have a great influence on the stability of the streams flowing through it and are among the important reasons for the aforementioned anomalous results. The relations between size of particles and velocities that will move them, given in most textbooks, do not apply even approximately to the case in hand. Moreover, it is not the average velocities near the bed and banks, as given in Fig. 3, that are to be reckoned with. Such "isovels", no doubt, were obtained by current meter and, therefore, represent mean velocities over a certain period of time. If an instantaneous velocity indicator, such as a Pitot tube or a Bentzel velocity tube, had been substituted for the current meter, momentary velocities greatly in excess (often two or three times) of the current meter velocity, would have been observed. For want of a better name the writer has termed them "stray currents." In glass-sided laboratory channels they may be observed in action. They are almost like, and have somewhat the effect of, a whip lash. It is these stray currents that move the bed material, rarely the mean velocity that can be measured with a current meter. These stray currents are induced by the irregularities of the channel bumps, holes, ripples, etc., and are phenomenon of turbulence.

If the Lacey formulas cited by the author are quite generally applicable, a stream flowing in its own alluvium would have the following remarkable properties:

(1) The cross-section on straight reaches tends to become semi-elliptical;

(2) The parameter of the ellipse (ratio of the major to one-half the minor axis or surface width to maximum depth) depends solely on the character of the silt as regards fineness;

---

[9] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[9a] Received by the Secretary February 24, 1936.

(3) For a given discharge the wet perimeter is constant and independent of the character of silt; and

(4) The silt factor bears a definite relation to the roughness coefficient in the Kutter and Manning formulas for discharge.

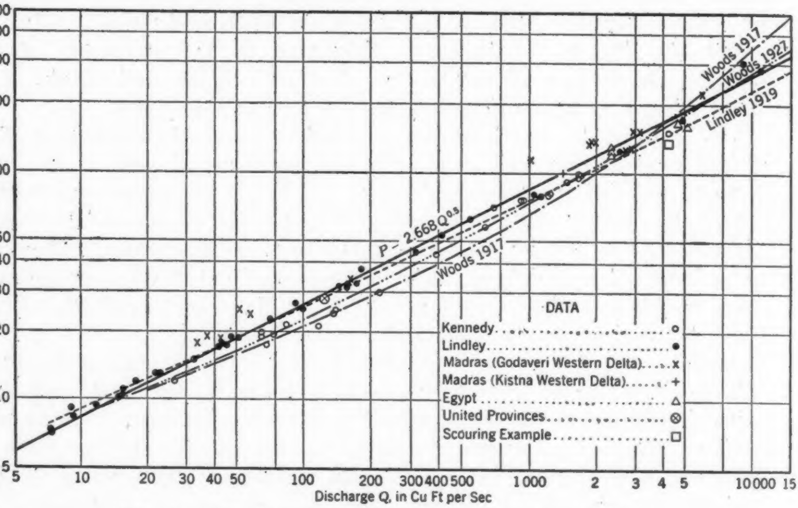


FIG. 4.—THE LACEY FORMULA, WITH SUPPORTING DATA.

Property (3) is expressed by the author's Equation (5). Fig. 4 is a reproduction of this formula from the Lacey paper[10]. Fig. 5 shows how certain alluvial channels not given by Lacey conform to this formula. The Imperial Valley canals do conform fairly well as credited by the author. The Rio Grande also conforms scatteringly. The Colorado River shows a splendid conformity to another law—not that of Lacey —as is also the case with the Huang-ho River.

It may be argued that the Lacey formula can only obtain where a given flow persists long enough to establish a stable regimen. The beds of the Colorado River and the Rio Grande

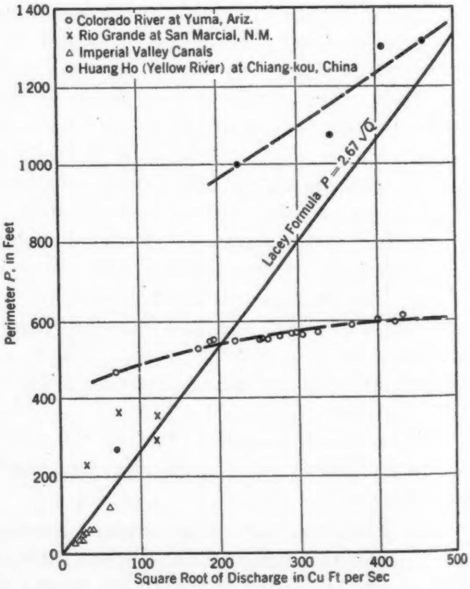


FIG. 5.—RELATION OF PERIMETER TO DISCHARGE. FOR ALLUVIAL STREAMS.

[10] "Some Problems Connected with Rivers and the Canals in Southern India", by J. M. Lacey, *Minutes of Proceedings*, Inst. C. E., Vol. 216, p. 150.

respond readily to changes in flow. Figs. 6 and 7 show cross-sections of both streams during 1929. Periods were selected when the discharge was fairly constant.

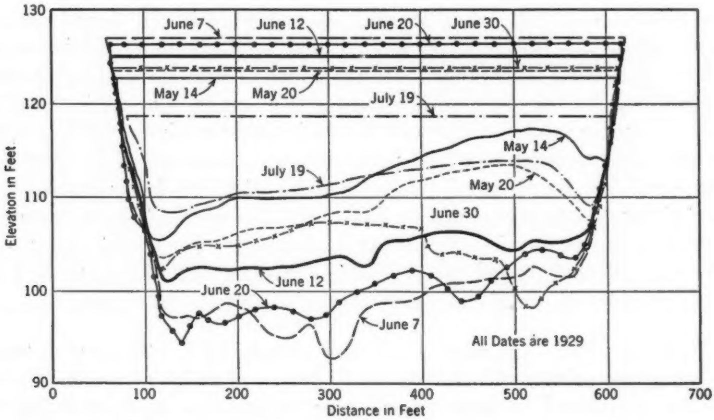


FIG. 6.—TYPICAL CROSS-SECTIONS, COLORADO RIVER AT YUMA, ARIZONA.

From data given by the author, particularly the range indicated by his Fig. 1, it is evident that the problem of designing canals in erodible material is an intricate and complex one in which the engineer's judgment and experience must be given a liberal measure of weight.

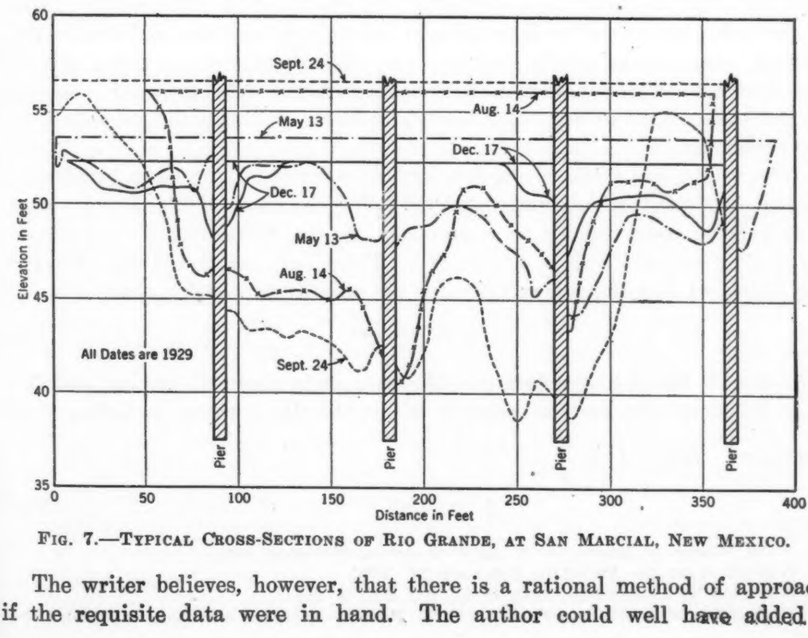


FIG. 7.—TYPICAL CROSS-SECTIONS OF RIO GRANDE, AT SAN MARCIAL, NEW MEXICO.

The writer believes, however, that there is a rational method of approach if the requisite data were in hand. The author could well have added a

fourth set of conditions to his list, namely, silt-laden canals requiring hydraulic elements such that silt will be carried through while not scouring the channel—the very condition under discussion.

Among the thousands of canals throughout the western part of the United States, which carry silt in their alluvial valleys, will be found some that meet this condition, and do carry silt without appreciable deposition or scour. The former Special Committee of the Society on Irrigation Hydraulics attempted to gather data on such canals and channels, but with only limited success. To secure data of value, field study and measurements are required for which the Committee had no funds. Suppose, however, that complete data were in hand for canals of all sizes and slopes that are known to carry their silt load through, and that complete data were also available as to the mechanical and physical properties of the silts so carried. Each such canal at once becomes an hydraulic model for the design of others to be constructed under similar conditions to which the principles of similitude will apply.

C. R. PETTIS,[11] M. AM. SOC. C. E. (by letter).[11a]—A clear and interesting discussion of the principles involved in the design of stable channels in erodible material is contained in this paper. Some statement as to the final types adopted for design would have been interesting.

Mr. Gerald Lacey, whose work is cited by Professor Lane, gave[12] certain equations from which the various elements of a stable channel in erodible material can be determined for any given value of discharge, Q; the various elements determined are the velocity, V, the cross-sectional area, A, the hydraulic radius, R, the width of stream, B, and the slope, S. He assumes that the stable channel has a semi-elliptical cross-section. The stable channel in the sense considered is in a straight reach, with uniform and steady flow, and of compact and fairly regular cross-section; if it is a canal it is of sufficient age to have adjusted itself to hydraulic conditions, and it is non-silting and non-scouring. Mr. Lacey's solution is unusually complete and comprehensive.

The writer has proposed an independent solution of the same problem, as applied to rivers[13], which was completed before he knew of Mr. Lacey's work. This solution was derived as a corollary to the width formula for floods: $Q \propto A^{1.25}$; from which $V \propto A^{0.25} \propto Q^{0.2}$. From data on the Miami River it was indicated that the proper value for the velocity in a stable channel is,

$$V = 0.8\, Q^{0.2} \qquad (11)$$

Attention is called to the fact that this is a main channel velocity, and that it is 2.5 times the average velocity of all the flood water, including overflow areas. Since $A = \frac{Q}{V}$,

$$A = 1.25\ Q^{0.8} \qquad (12)$$

[11] Col., Corps. of Engrs., U. S. A., Fort Hayes, Columbus, Ohio.

[11a] Received by the Secretary February 24, 1936.

[12] *Minutes of Proceedings*, Inst. C. E., Vol. 229, p. 259.

[13] "Relation of Rainfall to Flood Run-off", *Military Engineer*, March–April, 1936, p. 94.

From a Kennedy type of formula, assuming $C = 1$ for average conditions in alluvial sections of rivers, the maximum depth, d, of a cross-section will be,

$$d = V^{1.5} = 0.715\ Q^{0.3} \qquad (13)$$

The writer has investigated the shape of a natural stable channel which will best comply with known hydraulic conditions. The evidence indicates that the cross-section must be of the parabolic type indicated by $d \propto B^k$; that k is not less than 2 nor more than 3; and that $d \propto B^{2.5}$ is probably the ideal shape for a stable channel. A semi-ellipse is not an ideal shape for the channel of a natural stream. All the writer's formulas were derived from natural streams with drainage areas between 100 and 40 000 sq miles. For such streams (but not for smaller ones) the ratio, $\frac{B}{d}$, will be sufficiently great so that it may be assumed that B is equal to P, the wetted perimeter. Then, $\frac{R}{d}$ will be the author's form factor. The form factor for the parabolic curve, $d \propto B^{2.5}$, is 0.715; for this type of curve, $R = 0.715\ d$. Consequently,

$$R = 0.715\ d = 0.511\ Q^{0.3} \qquad (14)$$

$$B = \frac{A}{R} = 2.45\ Q^{0.5} \qquad (15)$$

and,

$$\frac{B}{d} = 3.43\ Q^{0.2} \qquad (16)$$

Substituting the foregoing values of V and R in the Manning formula, with Kutter's $n = 0.0225$, $S = 0.00036$.

The ideal stable river cross-section is a curve of the type, $d \propto B^{2.5}$. It is a characteristic of rivers that they are subject to very variable flows. No given cross-section can be theoretically perfect for more than one value of Q. A natural channel will adapt its shape to correspond to the bank-full value of Q. Since there is a certain range of stability, such a channel will be safe for a certain amount of overflow. If the overflows are frequent or high, the original shape may be modified slightly to meet this condition. When the water falls below bank-full stage, such a channel will remain within the limits of stability for flows at medium stages. At low stages there will be a tendency to silt along the sides, which may cause some irregularity in flow, and which may be followed by some scouring. These effects will tend to correct themselves with succeeding high stages. The ideal stable channel will remain stable for all ordinary conditions, except for frequent minor and temporary instability near the thalweg.

Considering a given section of a river in connection with a given value of Q, it is evident that V must not be less than a certain minimum value, or silting will occur; and V must not be more than a certain maximum value, or erosion will occur. River channels are carved out of the bed material by

erosion, and the writer's solution is based on the maximum value of V beyond which one cannot go and be certain of having a stable channel.

Mr. Lacey's formulas were derived largely from canals. Most of his values of Q were less than any the writer has considered, and his values of V are noticeably less than those of the writer. Mr. Lacey's solution represents the lower limit of safe velocities, which are primarily non-silting; and the writer's solution represents the upper limit of safe velocities, which are primarily non-scouring; and both solutions are necessary to an understanding of the problem as applied to rivers.

TABLE 2.—ELEMENTS OF STABLE RIVER CHANNELS IN ALLUVIUM

Description	Pettis (non-eroding)	Most stable	Lacey (non-silting)
Velocity, V	$0.8\,Q^{0.2}$	$0.8\,Q^{0.183}$	$0.8\,Q^{0.167}$
Area, A	$1.25\,Q^{0.8}$	$1.25\,Q^{0.817}$	$1.25\,Q^{0.833}$
Depth, d	$0.715\,Q^{0.3}$	$0.683\,Q^{0.317}$	$0.64\,Q^{0.33}$
Hydraulic radius, R	$0.511\,Q^{0.3}$	$0.488\,Q^{0.317}$	$0.468\,Q^{0.33}$
Surface width, B	$2.45\,Q^{0.5}$	$2.56\,Q^{0.5}$	$2.67\,Q^{0.5}$
Kutter s coefficient, n	0.0225	0.0225	0.0225
Slope, S	0.00036	$\frac{0.00038}{Q^{0.055}}$	$\frac{0.0004}{Q^{0.11}}$
Ratio, $\frac{B}{d}$	$3.43\,Q^{0.2}$	$3.75\,Q^{0.183}$	$4.17\,Q^{0.167}$

On the hypothesis that the Lacey solution and the writer's solution can be combined in the manner indicated, Table 2 gives the limits of stability for the various elements of a cross-section, for a given value of Q; and also intermediate values which are assumed to represent the most stable conditions. It is assumed that V_0 may vary from $d^{0.67}$ to $d^{0.5}$, or between $d = V^{1.5}$ (non-scouring) and $d = V^{2.0}$ (non-silting). This relationship was used in obtaining a non-silting value of d, since Mr. Lacey did not include d in his solution.

Table 3 indicates values of the elements corresponding to given values of Q. The values for $Q = 1$ indicates that the Lacey and the writer's solutions are consistent for a terminal condition; $Q = 1\,000$ is fairly representative of the original data used by Mr. Lacey; and $Q = 100\,000$ is representative of the writer's data.

TABLE 3.—ELEMENTS OF STABLE CHANNELS FOR GIVEN VALUES OF Q

Description	$Q = 1$		$Q = 1\,000$		$Q = 100\,000$	
	Pettis	Lacey	Pettis	Lacey	Pettis	Lacey
Velocity, V	0.8	0.8	3.18	2.53	8	5.45
Area, A	1.25	1.25	314	395	12 500	18 340
Depth, d	0.715	0.64	5.7	6.4	22.6	29.7
Hydraulic radius, R	0.511	0.468	4.1	4.7	16.1	21.7
Surface width, B	2.45	2.67	78	84	775	844
Kutter s coefficient, n	0.0225	0.0225	0.0225	0.0225	0.0225	0.0225
Slope, S	0.00036	0.0004	0.00036	0.00019	0.00036	0.00011
Ratio, $\frac{B}{d}$	3.43	4.17	13.7	13.2	34.3	28.5

The writer's formulas are a special solution for indicated conditions. Mr. Lacey's claims that his formulas are general. The two solutions have been checked with such river data as are available, and the foregoing explanation

seems to be the most satisfactory hypothesis under which one can co-ordinate the Lacey assumption that V varies as the sixth root of Q, and the writer's assumption that V varies as the fifth root of Q.

The laws governing velocity distribution in stream cross-sections are not definitely known, but to a certain extent they can be inferred from a study of available diagrams. The following statements appear to be justified: (1) In the stable parabolic cross-section, the bottom velocity will decrease gradually from the thalweg to the top of the banks; (2) the gravity component described by the author will increase gradually from the thalweg to the top of the banks; and (3) the result will be a condition of equilibrium at all points of the cross-section, except at low stages. It is obvious that a semi-elliptical cross-section will not meet the conditions for equilibrium near the top of the banks. The assumption of an elliptical cross-section does not materially affect the solution if the value of d is omitted.

The writer's conclusion is that Mr. Lacey's solution, to which the author refers, is fundamentally sound to the extent indicated, but that his theories should be slightly modified and expanded.

HARRY F. BLANEY,[13] M. AM. SOC. C. E. (by letter).[13a]—A problem is analyzed in this paper which has not always been given serious consideration, and the author presents data and conclusions which should prove of great value. Professor Lane has illustrated some of the problems which confront the designer when he attempts to apply formulas by Kennedy and others to conditions in the Southwestern United States.

In connection with silt investigations[14] made some years ago, several efforts were made by the writer and others to co-ordinate the movement of silt in the Lower Colorado River and in canals in Imperial Valley with the laws and formula held to be applicable to the movement of silt in other streams; but although there was agreement in some features, there was disagreement in others, so that, on the whole, few satisfactory conclusions could be drawn, largely because of the character of the silt and the chemical activity of certain salts in the waters of the river. The preponderance of fine silt held in suspension, and the fact that, in its movement downward or parallel to the grade of the channel, it seems to obey physical laws different from those governing bed silt, led to the conclusion that any formula applicable to one kind of silt would not apply to the other. Furthermore, it is not generally feasible to apply two sets of laws or formulas to the same part of a channel, since silt suspended at one time and place may become bed silt elsewhere at another time. Conversely, more or less bed silt may become suspended silt.

It was not difficult to trace the relation between the movement of the finer silt and the velocity of the current. All velocities in excess of about $\frac{2}{3}$ ft per sec transported the finer silt, not only in the river, but also in the canals.

[13] Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Los Angeles, Calif.

[13a] Received by the Secretary February 25, 1936.

[14] "Silt in the Colorado River and Its Relation to Irrigation", by the late Samuel Fortier, M. Am. Soc. C. E., and Harry F. Blaney, M. Am. Soc. C. E., *Technical Bulletin No. 67*, U. S. Dept. of Agriculture.

The chief difficulty arose in determining the velocities and other hydraulic elements that would cause the transportation of bed silt and the heavier grade of suspended silt.

In the earlier silt investigations, experiments were conducted with the object of applying Kennedy's formula (Equation (1)), to the canals of the Imperial Valley. Kennedy states[15];

"Strictly speaking, a separate value of V_0 should obtain at each season of the year, and for each class of sand carried in each reach of a canal, but this being impossible, the values of V_0 given must be taken as an all-round figure for what may be called the standard sand, found in the river beds near the hills. The question then arises how to define this sand, and a partial reply can be furnished, by taking as the criterion of coarseness, the rate of fall of the grains in still water. Thus, Grade 0.10 would consist entirely of grains which dropped at the rate of 0.10 feet per second; and Class $\frac{0.10}{0.15}$ would include atoms of all coarseness which fell between the rates of 0.10 and 0.15 feet per second."

Kennedy's method of grading silt is described by Parker[16]. A typical analysis is shown in Table 4(*a*).

TABLE 4.—GRADING OF DEPOSITS

(*a*) SIRHIND CANAL, PUNJAB (PERCENTAGE BY VOLUME)

Locality	0.00 / 0.10	0.10 / 0.20	0.20 / 0.30	0.30 / 0.40	0.40 / 0.60
Main Line:					
10 000 ft*......	4	50	30	9	7
15 000 ft*......	7	65	21	4	3
20 000 ft*......	5	53	28	7	7
Branch Line:					
150 000 ft*.....	28	56	12	4	0
170 000 ft*.....	29	57	11	3	0

(*b*) IMPERIAL CANALS, CALIFORNIA (PERCENTAGE BY WEIGHT)

Locality	0.000 / 0.105	0.105 / 0.207	0.207 / 0.339	0.339 / 0.500
Alamo Canal:				
1 mile†.......	33.5	49.3	8.1	7.0
Eastside Canal:				
53 miles†.....	97.1	2.7	0.2	0.0
75 miles†.....	70.7	27.5	1.5	0.0
88 miles†.....	95.1	4.5	0.0	0.0

* Distance below heading. † Distance below Rockwood Heading.

Attempts to grade the Colorado River silts by Kennedy's method were not very successful as the particles would cling together and produce eddies. However, by making standard sieve analyses by weight and then determining the rate of dropping, in feet per second, for particles of various size, some results were obtained. Compilations for two canals in the Imperial Valley are shown in Table 4(*b*). Although not directly comparable, the results indicate that the silt deposits in the Imperial Canal are somewhat lighter and finer in texture than those of the Sirhind Canal in India.

The author indicates that additional data are desirable on the "bed width-depth ratio." Table 5 has been compiled from original records of the U. S. Bureau of Agricultural Engineering (1918–1919) and may be of use in future studies.

[15] "Hydraulic Diagrams for Channels in Earth", by R. G. Kennedy, Second Edition.

[16] "The Control of Water", by the late Philip A Morley Parker, M. Am. Soc. C. E., Second Edition, p. 758.

TABLE 5.—BED WIDTH, DEPTH, AND MEAN, MAXIMUM, AND BED VELOCITIES FOR TYPICAL CANALS, IMPERIAL VALLEY, 1918–1919

Name of canal	Location	Bed width, in feet	Depth, in feet	Bed-width / Depth	Mean velocity, in feet per second	Maximum velocity, in feet per second	Bed velocity*, in feet per second
Date	Meter Bridge	14	1.5	9.3	2.66	2.95	2.3
Westside	At boundary	36	5.0	7.2	2.71	....	....
Lateral	Sharps Heading	4	1.0	4.0	1.58	1.74	1.0
Central	Sharps Heading	50	5.4	9.3	4.06	....	....
Brawley	El Centro Road	14	6.0	2.3	3.66	4.12	1.8
No. 5 Main	Yuma Road	23	4.8	4.8	3.52	4.25	2.1
No. 5 Main	Allison	30	4.7	6.4	4.04	5.10	2.5
Central	Boundary	38	5.5	6.9	2.80	3.20	1.7
Dogwood	Meter Bridge	19	3.2	5.9	1.67	1.95	0.9
Alamitos	Sharps Heading	24	3.3	7.3	2.54	2.75	1.9
Briar	Ten foot Drop	11	2.3	4.8	2.76	3.30	1.7
Evergreen	Dahlia Heading	10	1.5	6.7	1.56	1.88	0.9
Elder	Five Gates	10	3.1	3.2	3.00	3.40	2.2
Encino	Flume	22	3.4	6.5	3.48	4.00	2.2

* Estimated.

SIGURD ELIASSEN,[17] ASSOC. M. AM. SOC. C. E. (by letter).[17a]—The subject of stable channels in erodible material is of great interest to engineers in North China, where the silt problem is extremely serious and, consequently, channel stability is difficult to attain. Measurements taken throughout 1934 show that, where the Yellow River enters its diked section, it carried in suspension the stupendous total of 1 500 000 000 cu m, or 2 400 000 000 tons, of silt as this would have been deposited on the plain in a natural dry state. It was more than an average flood-year, but far from an exceptional one. Silt measurements were taken every day throughout the entire year and twice each day during the freshet season, when the silt percentage runs high. The silt load at the place mentioned ran as high as 19% by weight at times, and at places where the river is confined between high banks, such as at Shanchow, in Western Honan, even as high as 38 per cent. On the King Ho, in the Province of Shensi, a tributary to the Wei Ho, which again is a tributary to the Yellow River, or Huang Ho, the writer has personally measured 48% by weight—that is, weight of dried silt over weight of silt plus water. Similar high percentages have been measured on other tributaries of the Yellow River. If the customary procedure for determining silt percentages is used (namely, weight of dried silt over weight of clean water), the percentages can be greater than 100 as the weight of the silt becomes heavier than the weight of the water. It does not seem logical to express it in this manner.

The measurements are dependable enough and have been observed by too many engineers to be doubted. The writer has mixed artificial silt percentages in a gasoline tin and finds that they remain quite fluid, up to 50% by weight. At 56% the "solution" begins to be sticky, and between 58% to 60% it becomes a brick-maker's paste. When fully analyzed, the measurements for 1935 are likely to show similar results as regards silt transportation.

[17] Engr.-in-Chg., Survey Dept., The Yellow River Comm., Kaifeng, Honan, China.

[17a] Received by the Secretary on February 27, 1936.

The foregoing comments are offered only to demonstrate that American engineers have a "snap" compared to what confronts engineers in China when it comes to stability of river channels.

The author begins with the Kennedy rule for non-silting channels, $V_0 = C\ d^x$, and then reviews the work of other investigators who have included, also, the relation of width to depth, together with slope and silt character, in their formulas. He mentions that his investigations were prompted by the special conditions of the All-American Canal, for which it is claimed that the Kennedy rule does not apply. At any rate, considering the fine silt which this canal carries, surprisingly large constants have to be used to fit the Kennedy formula. Professor Lane, however, omits the hydraulic features of this canal, and the conclusions arrived at as regards its most satisfactory shape. The writer hopes that the author will include these in his closing discussion and also will show the character of silt in the form of an analysis curve, and the proportional quantity of suspended silt and "geschiebe", or bed load, carried by the canal. Professor Lane thinks that the varying quantity of suspended silt carried by the canal is a reason why the Kennedy rule does not apply. He also infers that the shape of the channel is a very important consideration when discussing stability.

Although the writer is in hearty agreement that channel shape is a very important factor, nevertheless, he thinks that the quantity and character of the silt transported as bed load is another highly important factor—more so than the varying quantity of the suspended silt, especially if this is fine. He has found that the Wei Pei Main Irrigation Canal, which has a slope of 1:2 330, a bed width of 6 m (19.7 ft), and side slopes of 1:1, carries fine sediment in suspension without silting the canal even when the load fluctuates from 0.1% to 10% by weight. Attempts have been made to use the canal when the silt ratio has been as high as 20 per cent. However, such attempts were discouraged not so much on account of silting of the main canal as the silting of the laterals. The silt is so fine that it all passes a 200-mesh sieve, the largest particles having a diameter of 0.03 mm. No coarse bed load is carried by this canal. It has been in use since 1933 and has not yet been cleaned out. The mean velocities in the canal, when it is running more than half-full, or depths from 1.5 m to 2.0 m (3.9 ft to 6.6 ft) is from 0.7 m to 1.0 m per sec. (2.3 to 3.3 ft per sec), and the discharge from 8.0 to 16 cu m per sec. (282 to 575 cu ft per sec); thus there is much fluctuation both in silt content and discharge.

From the foregoing it seems that as far as the All-American Canal is concerned there are other factors at work more important than silt variation, when considering the question of non-silting stability. As regards stability against scour the opinions expressed by the author seem very much to the point.

With regard to channel efficiency, a study of the roughness coefficient for the Yellow River shows that Kutter's n is fluctuating considerably at the same measuring section. Exactly what causes the fluctuation has not yet been fully discovered. The difficulties of securing good measurements for cross-section, velocity, and slope, under all conditions of flow and silt loads

are obstacles to reliable conclusions. On the Yellow River the variation of silt, shape of section, slope, velocity, and discharge are so marked that it ought to be possible to come to definite conclusions in time; but from measurements which have been taken, it is quite evident that n is not constant. One of the reasons why it varies, perhaps, is that bottom waves of fine sand pass the measuring station from time to time. During the passage of such sand waves bottom irregularities are set up which increase the frictional resistance and raise the value of n. At other times there are no bottom sand waves and the frictional resistance is less with a consequent lowering of n. It is difficult to arrive at any conclusion as regards changes in n, due to a varying silt percentage.

Scour and re-fill of the bed or cross-sectional changes which proceed so rapidly on the Yellow River are, on the other hand, factors which definitely seem to influence n. This leads the writer to think that bed irregularities which slow up the velocity and raise the water level may cause deposits to occur, and he believes that the bed-load movement will create sufficient irregularities to cause deposition especially if the actual velocities under an ideal canal condition lie close to the silting velocities. On the other hand, under a heavy suspended silt load the coarser bed-load silt grains should be carried along more easily, since they are "buoyed up" by the finer silt particles in suspension, due to the effect of increased specific gravity which mainly affects particles coarser than the average.

With regard to the Kennedy formula the writer wishes to draw attention to feet and metric units. This seems very elementary; but recently he has several times encountered cases in which metric units have been introduced directly into the common form of Kennedy's formula for the Bari Doab System, $V_0 = 0.84\, d^{0.64}$, which is in English units. In metric units, this formula reads, $V_0 = 0.56\, d^{0.64}$.

In logarithmic form, the Kennedy formula reads:

$$\log V_0 = \log C + n \log d \qquad (17)$$

As far as can be judged, Kennedy plotted his results on logarithmic paper. The points fell rather scattered and he drew the best average straight line and obtained his constants from Equation (17). It is interesting to note that the ordinary discharge or velocity formula is also of this form when the gauge height, H, is substituted for the depth, d. Assuming cases in which the Kennedy rule is applicable, the non-silting channel should be one in which the actual velocities and the Kennedy velocities coincide, or better, the actual velocity curve lying a short distance to the right of, but parallel to, the Kennedy curve. In practice, however, the two curves seldom coincide, but cross each other at some definite velocity.

If computed velocity curves are plotted on logarithmic paper for various kinds of sections (such as triangular, trapezoidal, or rectangular sections), it will be found that the slope of the curves varies somewhat with the shapes and dimensions of the sections, especially when these sections are narrow. As the sections widen the curves become parallel. Hence, it is impossible to supply one set of Kennedy coefficients even to one irrigation system.

In order to compare the Kennedy formula with the Chezy-Kutter formula the latter may also be expressed in logarithmic form:

$$\log V = \log C + \frac{1}{2} \log S + \frac{1}{2} \log R \quad \text{.............} (18)$$

Considering, first, the Chezy-Kutter formula, it seems evident that since the slope is constant for a given section of a canal, $\frac{1}{2}$ log S becomes a part of the intercept constant. If there is any change in S, the curve will be displaced parallel to itself.

Coefficient C is a function of R and also of the roughness constant, n; but C varies in the same manner throughout a considerable range in R-values for almost any value of n, and the effect on the curve due to a change in n is to displace it nearly parallel to itself. In other words, a change in n has little effect in changing the slope of the velocity curve. For the most part it changes the intercept value of the curve; but since C has also been made a function of R, it too should influence the slope of the curve.

Strictly, Coefficient C should be regarded as being made up of two parts, a constant curve intercept part and a curve slope factor. The constant intercept part, therefore, may be considered as being due to channel roughness, which is fixed within narrow limits, provided there is no bed-load movement and the channel keeps its shape otherwise. The slope factor part of C, for clear water, would then be a cross-section form factor which would change according to the shape of the cross-section. In the Chezy-Kutter formula, C has been made a variable depending to a certain extent upon R and then a general, fixed exponential factor, 0.5, made to represent the remainder of the form factor. To the writer it would have seemed more logical to have given C a fixed value according to the value of n, and then to have had an exponent of R which would change according to the shape of the cross-section. The Manning and Forschheimer formulas involve only n; but they also fix the exponent of R. It is true that form factors for areas of different shapes do not vary much, but it seems obvious that an improvement would result from making the exponent of R vary according to the shape of the section.

In the Kennedy formula, the disturbing element, silt, is introduced. Proceeding through the same line of argument it becomes obvious that the silt composition will affect C, or the intercept constant. Unfortunately, the silt composition is subject to change since floods from different tributaries may bring in different grades of silt. This is very noticeable on rivers in North China. If the silt composition is such that a part is carried as a bed load, this fact is likely to affect the roughness and disturb C still further. It may also affect the shape of the channel and thus change the exponential factor, d^n, which unquestionably can be regarded as a cross-section form factor. If the silt composition does not change much for varying silt percentages, then, up to the point where silting begins, the silt will be carried vertically and longitudinally very much in the same manner throughout the section, even when the water level fluctuates considerably, due to a varying discharge, and the channel is likely to have the same form factor and hence

a constant exponent of d. In such cases both C and n can be determined reliably; but the difficulty in applying the Kennedy rule begins when the silt composition changes and a heavy "geschiebe" is introduced. For such cases it may be possible to determine the upper limit for C; but to find a reliable value for n is more difficult since anything may happen to the shape of the section when increasing the velocity to an extent which will make sure that the "geschiebe" is being transported without silting the canal. Most likely, in such cases, there will have to be desilting works. This seems to have been the case with the All-American Canal.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TRUSS DEFLECTIONS: THE PANEL DEFLECTION METHOD

Discussion

By Fang-Yin Tsai, Assoc. M. Am. Soc. C. E.

Fang-Yin Tsai,[18] Assoc. M. Am. Soc. C. E. (by letter).[18a]—The method for computing truss deflections presented in this paper is certainly somewhat simpler and more convenient than the elastic weight methods of either the bar changes developed by Professors Mohr and Müller-Breslau, or the angle changes (also known as bar-chain method) developed by Professor Land, which, as far as the writer knows, have been the only analytical methods available to date for computing the truss deflections for all the panel points in any chord by a continuous operation. The author is to be commended for his ingenuity.

It is regrettable, however, that the derivation of Equations (4) and (5) was not included in order to clarify their application and their limitation. These formulas are valid only for trusses having vertical web members and equal panel lengths. It is possible to modify the method, expressing it in a more general form, so as to make it applicable to trusses of any type.

Fig. 16 shows an intermediate oblique panel, MN_{mn}, of a truss, with M and N denoting the upper panel points and m and n the corresponding lower panel points. The dimensions of the panel are designated by P, p, Y, y, h, and k, which will be considered as positive when measured upward from, or to the right of, a certain reference point (say, M, Fig. 16) and otherwise negative. The lengths of the five bars of the panel are designated, respectively, by L_{MN}, L_{mn}, L_{Mm}, L_{Nn}, and L_{Nm}. Let ΣR be the sum of the rotations of all the deformed panels to the left of this panel, ΣR being considered as positive when it is clockwise and negative when counter-clockwise. Considering the panel point, M, as the reference point (that is, considering M as fixed in position after it has been deflected by all the deformed panels to its left), the panel, as a fixed frame, will first be rotated about M through the angle, ΣR.

Note.—The paper by Louis H. Shoemaker, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1936, by Messrs. David B. Hall, and E. Mirabelli; February, 1936, by Messrs. William Bertwell, and Robert H. Hurlbutt; and March, 1936, by Messrs. A. A. Eremin, T. P. Noe, Jr., David A. Molitor, and Glenn L. Enke.

[18] Prof. of Structural Eng., Dept. of Civ. Eng., National Tsing Hua Univ., Peiping, China.

[18a] Received by the Secretary February 10, 1936.

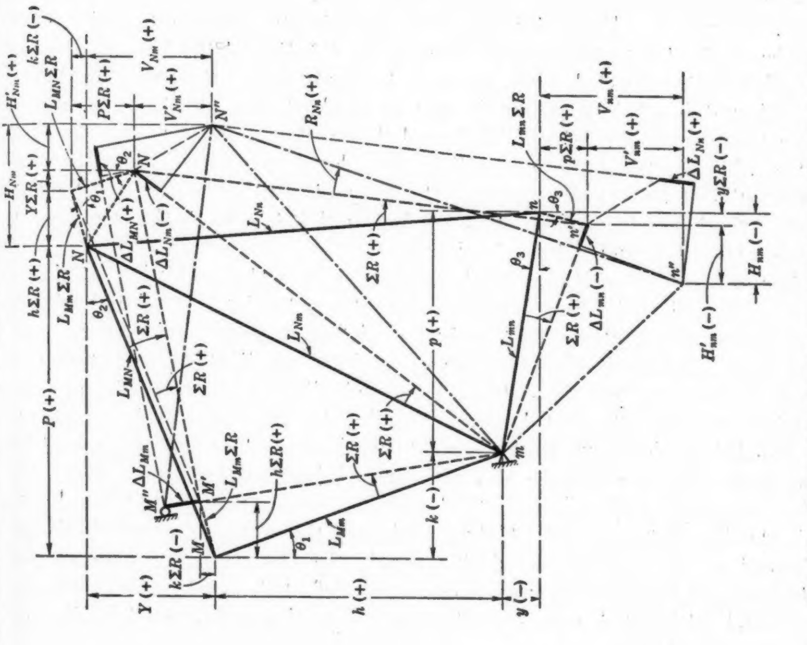


FIG. 17.—PANEL DEFLECTION DIAGRAM WITH LOWER PANEL POINT *m* AS THE REFERENCE POINT.

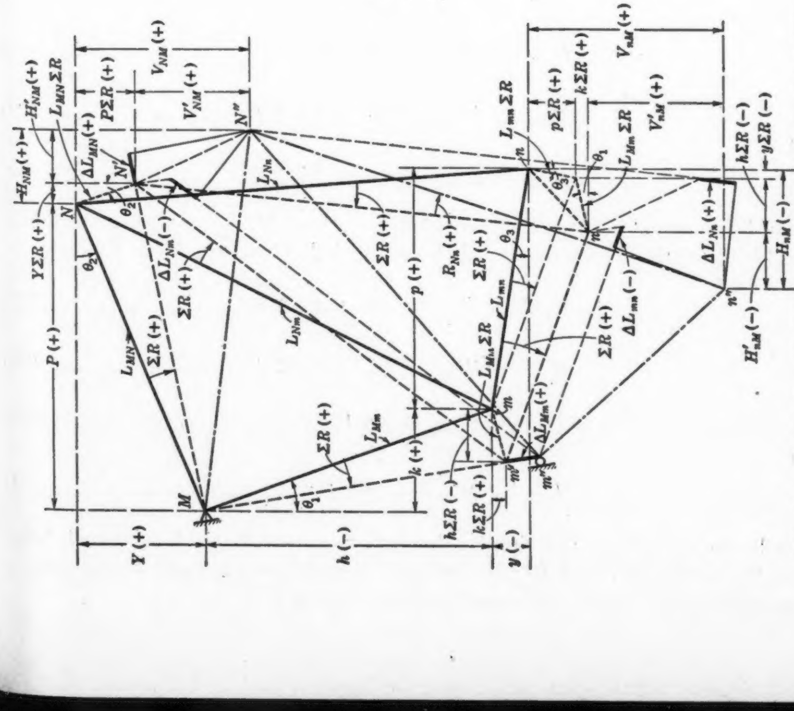


FIG. 16.—PANEL DEFLECTION DIAGRAM WITH UPPER PANEL POINT *M* AS THE REFERENCE POINT.

Thus, the panel points N, n, and m, are deflected, respectively, to the positions of N', n', and m'. The deflection, NN', is evidently equal to $L_{MN} \Sigma R$, which may be resolved into the vertical component, $P \Sigma R$, and the horizontal component, $Y \Sigma R$. The deflection, nn', may be resolved into the two components, $L_{mn} \Sigma R$ and $L_{Mm} \Sigma R$, which again may be resolved, respectively, into the two vertical components, $p \Sigma R$ and $k \Sigma R$, and the two horizontal components, $y \Sigma R$ and $h \Sigma R$. All the deflections will be considered as positive when they are downward from, or to the right of, the original position of the panel point. Assume the bar deformations ΔL_{MN}, ΔL_{Mm}, and ΔL_{Nn}, to be the increases in length, and ΔL_{Nm} and ΔL_{mn}, the decreases..

Again, considering M as a fixed reference point and, in addition, Mm' as a reference bar, fixed in direction, the deflected panel points, N', n' and m', due to those bar deformations, will be further deflected, respectively, to the positions of N'', n'', and m'', which are located by the Williot construction as shown in Fig. 16. Let V'_{NM}, H'_{NM}, V'_{nM}, and H'_{nM} be the vertical and horizontal components of the deflections, $N' N''$ and $n' n''$, respectively, which are the panel deflections due to the bar deformations of the panel alone, and let V_{NM}, H_{NM}, V_{nM}, and H_{nM} be the vertical and horizontal components of the deflections, NN'' and nn'', respectively, which are the panel deflections due to both the bar deformations of the panel and the total rotation of all the deformed panels to the left of this panel. In the foregoing notation for deflections, the first subscript denotes the deflected panel points, whereas the second subscript denotes the reference point. From Fig. 16 the following relations are evident:

$$V_{NM} = V'_{NM} + P \Sigma R \qquad (27)$$

$$H_{NM} = H'_{NM} + Y \Sigma R \qquad (28)$$

$$V_{nM} = V'_{nM} + (p + k) \Sigma R \qquad (29)$$

and,

$$H_{nM} = H'_{nM} + (y + h) \Sigma R \qquad (30)$$

If the lower panel point, m, is considered as the reference point, as shown in Fig. 17, the various components of the panel deflections will be as follows:

$$V_{Nm} = V'_{Nm} + (P + k) \Sigma R \qquad (31)$$

$$H_{Nm} = H'_{Nm} + (Y + h) \Sigma R \qquad (32)$$

$$V_{nm} = V'_{nm} + p \Sigma R \qquad (33)$$

and,

$$H_{nm} = H'_{nm} + y \Sigma R \qquad (34)$$

In order to represent, very clearly, the various components of the panel deflections, the total rotation, ΣR, and the bar deformations are much exaggerated in proportion to the size of the panel in Figs. 16 and 17.

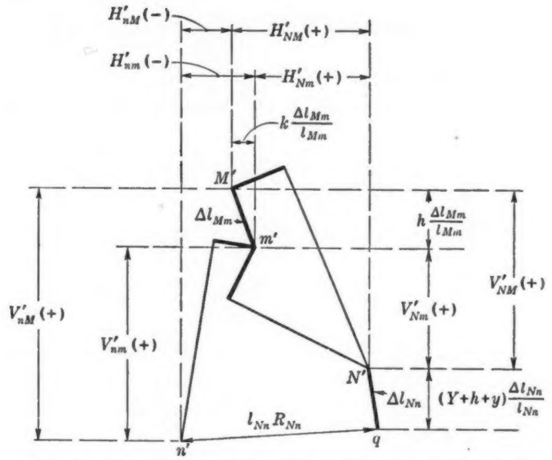


FIG. 18.—WILLIOT DEFLECTION DIAGRAM FOR THE PANEL SHOWN ON FIG. 16 AND FIG. 17.

From Fig. 18, which shows the Williot diagram constructed for both the two cases shown in Figs. 16 and 17, the following relations are evident:

$$V'_{NM} = V'_{Nm} + h\,\frac{\Delta\,L_{Mm}}{L_{Mm}} \qquad (35)$$

$$V'_{nM} = V'_{nm} + h\,\frac{\Delta\,L_{Mm}}{L_{Mm}} \qquad (36)$$

$$H'_{NM} = H'_{Nm} + k\,\frac{\Delta\,L_{Mm}}{L_{Mm}} \qquad (37)$$

$$V'_{nM} = H'_{nm} + k\,\frac{\Delta\,L_{Mm}}{L_{Mm}} \qquad (38)$$

$$V'_{nM} = V'_{NM} + (Y + h + y)\,\frac{\Delta\,L_{Nn}}{L_{Nn}} \qquad (39)$$

and,

$$V'_{nm} = V'_{Nm} + (Y + h + y)\,\frac{\Delta\,L_{Nn}}{L_{Nn}} \qquad (40)$$

In Equations (35) to (40), the dimensions, h, k, Y, and y, have numerical values only, and $\Delta\,L_{Mm}$ and $\Delta\,L_{Nn}$ will be positive when they are increased in length and negative when decreased. In the case of a rectangular panel, with vertical members, $Y = y = k = 0$, $P = p$, and $h - L_{Mm} = L_{Nn}$, Equations (27) to (40) will be much simplified. These formulas will suffice to indicate the fact that the values of the panel deflections, V, H, V', and H', may be altogether different in accordance with the reference point chosen and, for the sake of clearness, the latter should be preferably indicated in computing the former.

In Fig. 18, note also that $n'q = L_{Nn} R_{Nn}$, or $R_{Nn} = \frac{n'q}{L_{Nn}}$, which is the rotation of the panel due to its bar deformations, with Mm as the reference bar. If Bar Nn is a vertical member, the rotation, R_{Nn}, will be,

$$R_{Nn} = \frac{H'_{NM} - H'_{nM}}{L_{Nn}} = \frac{H'_{Nm} - H'_{nm}}{L_{Nn}} \qquad (41)$$

When both Nn and Mm are vertical, $H'_{NM} = H'_{Nm}$; and $H'_{nM} = H'_{nm}$, then,

$$R_{Nn} = \frac{H'_{NM} - H'_{nm}}{L_{Nn}} \qquad (42)$$

Since the total rotation, ΣR, and the bar deformations, are always very small in comparison with the dimensions of the panel, the computation of the panel deflections, V' and H', due to its bar deformations may be made without including the effect of ΣR.

For the computation of the panel deflections, V', H' (which correspond to the author's V_1 and H_1), and the rotation, R, the author has presented Equations (1), (2), and (3), respectively, which are valid only for a panel with vertical members. It seems to the writer that those formulas are likely to cause confusion in their application, and this is indicated by the fact that a special column is provided in Tables 1 and 2 of the paper to "symbolize" each bar in applying the author's formulas, even to the simple examples. Therefore, for such computations, the writer would prefer the method of work, or even the Williot diagram, the latter being particularly expedient for the oblique panel shown in Figs. 16 and 17. With the construction of the simple Williot diagram shown in Fig. 18, all the eight panel deflections, V' and H', and the rotation, R, of the panel are obtained at once. It has been universally recognized that the Williot diagram is the best method when the deflections of all the panel points of a truss are desired, the only disadvantage being that the diagram increases in size, rapidly, as more panels are included in the construction, and, consequently, it is somewhat difficult to obtain the results with a high degree of precision. However, if applied to one panel only, the Williot diagram will have no disadvantage. By combining the Williot diagram for computing V', H', and R, and the author's method for summing up the panel deflections to obtain the deflection of the truss, a semi-analytical method is evolved which, perhaps, will be the best method of all.

With the panel deflections known, it will be very simple to compute the total deflections of the truss. Fig. 19 shows a non-deformed truss, with its deformed position, the end panel point, a, being a reference point and the bar, ab, a reference member, fixed in direction. The total deflection for any panel point is designated by the second subscript, a, which indicates the reference point; thus, V_{Da} is the total vertical deflection of the Panel Point D with Panel Point a as a reference point. From Fig. 19 it is evident that the total deflection of any panel point in any chord is the algebraic sum of the panel deflections of all the panel points to the left of, and including, the panel point in question.

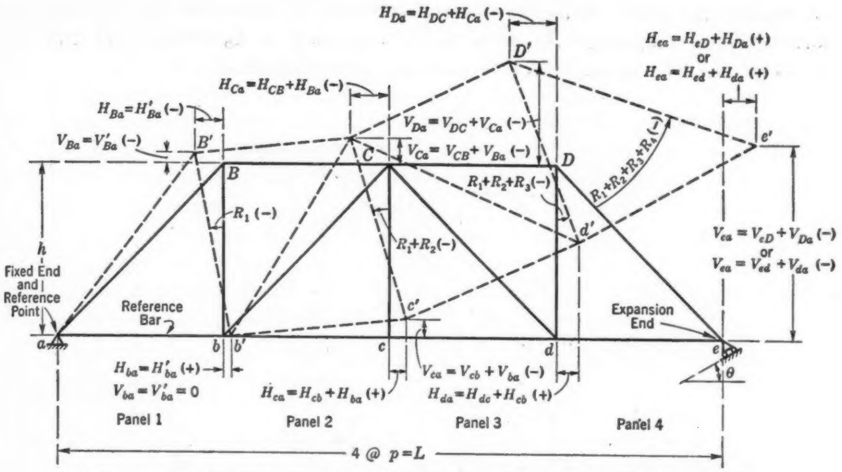


FIG. 19.—RELATIONS OF TOTAL DEFLECTIONS WITH PANEL DEFLECTIONS.

The correctness of this statement may be proved by the following example, considering Panel Point C. Since V_{CB} is the vertical panel deflection of C, with B as the reference point, and if Panel Point B also has its own vertical deflection, V_{Ba}, with a as the reference point, the algebraic sum, $V_{CB} + V_{Ba} = V_{Ca}$, must be the total vertical deflection of C with a as the reference point. Of course, all these deflections are computed with the bar, ab, as the reference bar. It should be observed that in summing up the panel deflections, care must be taken regarding their reference points and, by using panel deflections with different reference points, the total deflection of any panel point may be obtained in various ways. Thus, the total vertical deflection, V_{da}, of the panel point, d, may be obtained in any of the following four ways:

$$V_{da} = V_{dc} + V_{cB} + V_{Ba} \quad (43)$$

$$V_{da} = V_{dc} + V_{cb} + V_{ba} \quad (44)$$

$$V_{da} = V_{dc} + V_{cb} + V_{ba} \quad (45)$$

or,

$$V_{da} = V_{dc} + V_{cB} + V_{Ba} \quad (46)$$

The significance of Equations (43) to (46) is evident from Fig. 20, which shows the various panel deflections for different reference points and also the total deflections (shown barred; thus, $\overline{V_{ca}}$) with a as the reference point, for all the panel points of the truss in Fig. 19.

Since the computation of the total deflections is merely a matter of summation as soon as the panel deflections are known and since the summation may be somewhat varied according to the choice of the reference points on the basis

of which the panel deflections are computed, it seems to the writer that formulas for computing the total deflections such as Equations (4) and (5) of the paper are not only unnecessary, but also undesirable.

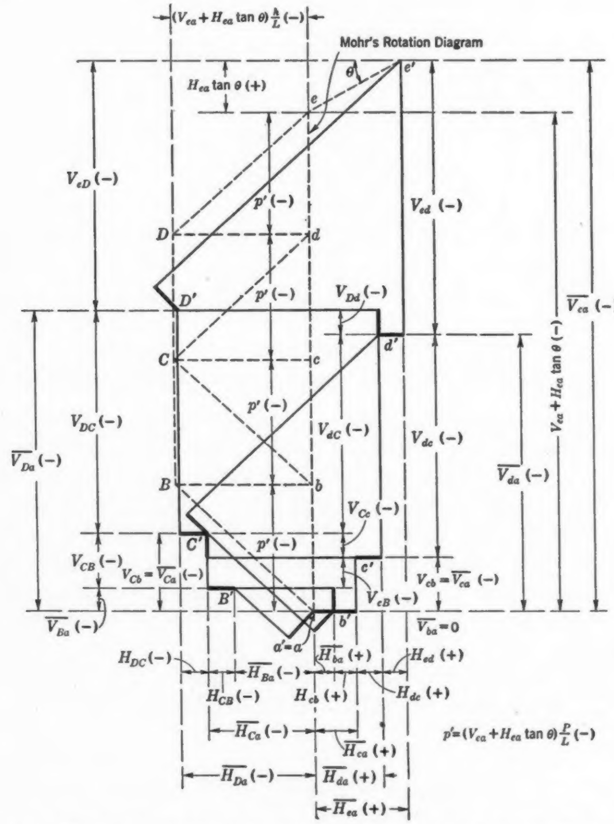


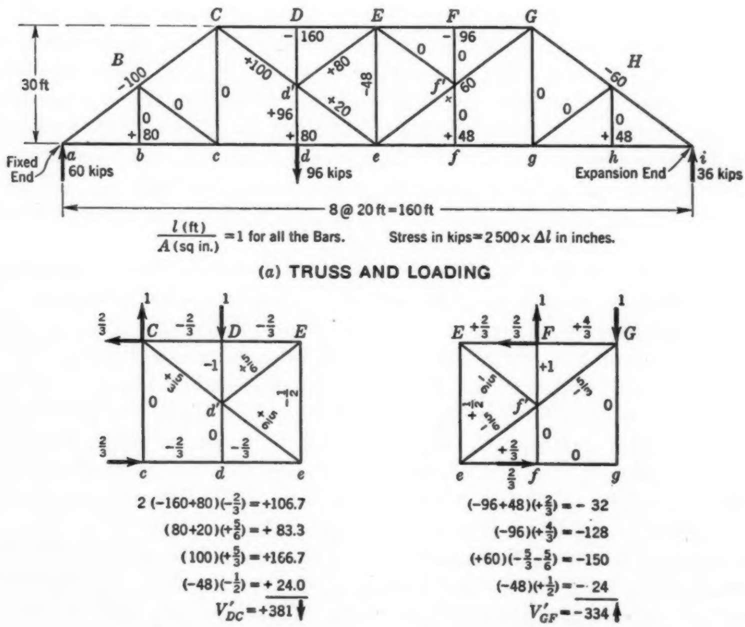
FIG. 20.—WILLIOT-MOHR DIAGRAMS SHOWING THE VARIOUS DEFLECTION COMPONENTS WITH DIFFERENT REFERENCE POINTS FOR THE TRUSS OF FIG. 19.

The author has presented two numerical examples, in which both the trusses and their deformations are symmetrical with respect to the center lines. In computing the panel deflections, the center panel point and the center bar are taken as references (that is, they are assumed to be fixed) and, by so doing, the true deflections can be obtained by merely transposing the reference point from the center panel point to the end panel point, which is actually fixed. This procedure is certainly advantageous in these symmetrical cases. However, in cases in which either the truss or its deformations are asymmetrical, such a procedure will require corrections both for transposing the reference point and rotating the reference bar in order to obtain the true deflections from the total deflections, and the computations might become seriously com-

plicated in an analytical method such as this. Therefore, in asymmetrical cases, it will always be advantageous to use the end panel point, (preferably the left end) and a convenient bar connected to it, for reference.

In this manner the computations may proceed from panel to panel toward the right, and only corrections for rotating the reference bar will be required to obtain the true deflections. Such corrections for rotation are similar to the Mohr rotation diagram in Williot's construction and are also very simple if applied analytically to this method. Fig. 20 shows the Williot and the Mohr diagrams for the truss of Fig. 19, from which the following general rule may be stated for obtaining the true deflections from the total deflections in a truss with its expansion end supported by rollers on any inclined plane, making an angle, θ, with the horizontal:

From the total panel deflection of any panel point subtract algebraically the quantity, $\dfrac{V_{ea} + H_{ea} \tan \theta}{L}$, multiplied by its vertical distance from Reference Point a when computing the horizontal deflection and by its horizontal distance from Reference Point a when computing its vertical deflection.



(a) TRUSS AND LOADING

(b) COMPUTATION OF V'_{DC} (c) COMPUTATION OF V'_{GF}

FIG. 21.

The vertical or horizontal distance of any panel point from Reference Point a will be positive when the panel point in question lies, respectively, above, or to the right of, a. The proper sign must also be assigned to V_{ea} and H_{ea}, which are, respectively, the total vertical and horizontal deflections

of the expansion end (Panel Point e, Fig. 19). When the plane that supports the rollers is horizontal (that is, when $\theta = 0$), the quantity, $\frac{V_{ea} + H_{ea} \tan \theta}{L}$, is reduced to $\frac{V_{ea}}{L}$. Applying the foregoing general rule to the example of Figs. 19 and 21, it is noted that: (1) The horizontal deflections of all the lower panel points require no corrections; (2) for the horizontal deflections of all the upper panel points, the quantity, $(V_{ea} + H_{ea} \tan \theta) \frac{h}{L}$, must be subtracted, algebraically from the respective total horizontal deflection; and (3) for the vertical deflections of all the upper and lower panel points, the quantity, $(V_{ea} + H_{ea} \tan \theta) \frac{p}{L}$, multiplied by n, the number of panels which the panel point in question is away from Reference Point a, must be subtracted algebraically from the respective total vertical deflection.

For a subdivided panel containing a quadrilateral such as that in the Pettit truss, it is evident that the computation of the panel deflections can not be made by considering the panel only, since it is unstable in itself. To obtain the correct results, the main panel containing such a quadrilateral must be considered as a free body in computing panel deflections, but the dummy unit load and its reactions are still considered as acting on the subdivided panel in question. This is clearly illustrated in Figs. 21(b) and 21(c).

In this case, again, it is noted that, for computing the panel deflections, V' and H' formulas of limited applicability, such as Equations (1) and (2) of the paper, will be of little value and it will also be futile to attempt devising such equations in a generally valid form.

In applying a method of this nature, systematic tabulation of computations and consistent convention for signs are both of the utmost importance; otherwise, confusions and errors are likely to occur. Even for the very simple example given in the paper, the writer experienced some difficulty at first in following Tables 1 and 2. For instance, the author has not indicated the reference points for the vertical deflections in Table 1, Column (7), so it is somewhat difficult to understand how they can be added in Column (11). The value, 1 355, in Column (11) is evidently the numerical sum of the values, −354, −267, and +734 in Column (7), indicating, obviously, that the signs of the latter have been disregarded.

In order to illustrate the application of the method to an asymmetrically deformed truss with subdivided panels, and also to show the method of tabulating the computations without using Equations (4) and (5) (as the writer prefers), the example[19] shown in Fig. 21(a) is completely developed and tabulated systematically in Table 11. The deflections, V' (Step (3)), for all the panel points in the upper chord, and H' (Steps Nos. (12) and (19)), for all the panel points in both the chords are first computed and tabulated, most of the latter being merely the bar deformations of the chord members.

[19] The example is taken from "Structural Theory", by H. Sutherland and H. L. Bowman, Members, Am. Soc. C. E., John Wiley & Sons, New York, Second Edition, 1935, p. 173, Figs. 7 to 12.

TABLE 11.—COMPUTATIONS FOR THE DEFLECTIONS OF ALL THE PANEL POINTS OF THE TRUSS SHOWN IN FIG. 21(*a*).
(With Panel Point *a* as Reference Point and Bar *ab* as Reference Bar).

Quantity	Step No.	Upper and Lower Panel Points: *B–b*	*C–c*	*D–d*	*E–e*	*F–f*	*G–g*	*H–h*	*i*
R (of the preceding panel)	(1)	....	*Bb* −13.7	*Cc* − 9.5	*Dd* − 8.0	*Ee* − 8.0	*Ff* − 4.8	*Gg* − 4.8	*Hh* − 5.7
Σ R	(2) = Σ (1)		−13.7	−23.2	−31.2	−39.2	−44.0	−48.8	−54.5
Vertical Deflections, Upper Panel Points									
V'	(3)	*Ba* 0	*CB* − 107	*DC* + 381*	*ED* − 186	*FE* − 74	*GF* − 334†	*HG* − 50	*iH* − 164
PΣ *R* (*P* = 20 ft.)	(4) = 20 × (2)	0	− 274	− 464	− 624	− 784	− 880	− 976	−1 090
V = *V'* + *P* Σ *R*	(5) = (3) + (4)	0	− 381	− 83	− 810	− 858	−1 214	−1 026	−1 254
Total, *V* = Σ *V*	(6) = Σ (5)	*Ba* 0	*Ca* − 381	*Da* − 464	*Ea* −1 274	*Fa* −2 132	*Ga* −3 346	*Ha* −4 372	*ia* −5 626
Correction for rotation	(7)	703‖	−1 407	−2 110	−2 813	−3 516	−4 220	−4 923	−5 626
True *V*	(8) = (6) − (7)	*B* + 703	*C* +1 026	*D* +1 646	*E* +1 539	*F* +1 384	*G* + 874	*H* + 551	*i* 0
True *V* × 0.0004 (in.)	(9)	+0.28	+0.41	+0.66	+0.62	+0.55	+0.35	+0.22	0
Vertical Deflections, Lower Panel Points									
Δ*L*, in inches	(10)	0	0	*d' d* +0.04	*Ee* −0.02	0	0	0	0
True *V* × 0.0004 (in.)	(11) = (9) + (10)	*b* +0.28	*c* +0.41	*d* +0.70	*e* +0.60	*f* +0.55	*g* +0.35	*h* +0.22	*i* 0
Horizontal Deflections, Upper Panel Points									
H'	(12)	*Ba* − 125	*CB* − 205	*DC* − 160	*ED* − 160	*FE* − 96	*GF* − 96	*HG* − 38	*iH* + 48
Y Σ *R* ‡	(13)	0	− 205	0	0	0	0	+ 732	+ 817
H = *H'* + *Y* Σ *R*	(14) = (12) + (13)	− 125	− 410	− 160	− 160	− 96	− 96	+ 694	+ 865
Total *H* = Σ *H*	(15) = Σ (14)	*Ba* − 125	*Ca* − 535	*Da* − 695	*Ea* − 855	*Fa* − 951	*Ga* −1 047	*Ha* − 353	*ia* + 512
Correction for rotation	(16)	− 527‖	−1 054‖	−1 054	−1 054	−1 054	−1 054	− 527	0
True *H*	(17) = (15) − (16)	*B* + 402	*C* + 519	*D* + 359	*E* + 199	*F* + 103	*G* + 7	*H* + 174	*i* + 512
True *H* × 0.0004 (in.)	(18)	+0.16	+0.21	+0.14	+0.08	+0.04	+0.00	+0.07	+0.21
Horizontal Deflections, Lower Panel Points									
H = *H'* = Δ *L*	(19)	*ba* + 80	*cb* + 80	*dc* + 80	*ed* + 80	*fe* + 48	*gf* + 48	*hg* + 48	*ih* + 48
True *H* = Total *H* = Σ *H*	(20) = Σ (19)	*b* + 80	*c* + 160	*d* + 240	*e* + 320	*f* + 368	*g* + 416	*h* + 464	*i* + 512
True *H* × 0.0004 (in.)	(21)	+0.03	+0.07	+0.10	+0.13	+0.15	+0.17	+0.19	+0.21

* See the computations in Fig. 21 (*b*). † See the computations in Fig. 21 (*c*). ‡ $Y = +15$ ft for Panel *CB*; -15 ft for Panels *HG* and *iH*; and 0 for all the remaining panels. ‖ $-703 = +20 \times \frac{-5\,626}{160}$; $-527 = +15 \times \frac{-5\,626}{160}$; and $-1\,054 = +30 \times \frac{-5\,626}{160}$

The rotations, R, are computed by Equation (41) or Equation (42). With those values known, the remaining computations are merely a matter of arithmetic, the operations for obtaining the results in every row being also indicated. It may be noted that the true vertical deflection of any lower panel point (Step No. (11)) is equal to the algebraic sum of the same deflection of its corresponding upper panel point (Step No. (9)) and the bar deformation of the vertical member connecting the two panel points (Step No. (10)). This relation is always valid for trusses having vertical members, irrespective of the shape of the chords and the arrangement of the diagonals. The exact agreement between the values (+0.21 in.) of the true horizontal deflection of Panel Point i computed from both the upper and the lower chords serves as a check.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

WIND STRESSES IN REINFORCED CONCRETE ARCH BRIDGES

Discussion

BY LEON BLOG, ASSOC. M. AM. SOC. C. E.

LEON BLOG,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—An elegant method of analyzing an arch subjected to lateral stresses is presented in this paper which the author has explained by an application to a reinforced concrete arch loaded by horizontal wind forces. This method is elegant because it weaves together the theorems of Castigliano and Maxwell, but it is not so simple as the method devised by Professor Maurice Levy[7] who solved the problem by a graphic solution based upon a rigorous mathematical analysis. By his method an arch can be solved in one day at as many sections as desired and, furthermore, the section of the arch at which a strut subject to the least bending or torsion should be located, aside from the question of æsthetics, is clearly indicated. The author's method is not that speedy nor can the physical behavior of the arch be visualized without considerable work supplementary to the analysis for various sections.

Arch Ribs Without Bracing.—Although the introduction of so many types of moments is perfectly natural due to the use of the Castigliano and Maxwell theorems, the visualization of the contribution of each toward the solution of the problem is difficult. The use of the designation, "deformation", for the values, θ and τ, is confusing. Although an angular change in the position of a section is, in a sense, a deformation, the ordinary use of the term is to describe changes of length. It is only when the reader comes to the paragraph following Table 1 that he learns that θ and τ are angular changes unless he happens to recall that the basis of the terms in the parentheses of Formula (2) (which leads to the internal work, W) is that the angular change at any section of a member subjected to bending is the derivative of the inter-

NOTE.—The paper by A. A. Eremin, Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Asst. Chf. Structural Engr., Div. of Bridges and Structures, City of Los Angeles, Los Angeles, Calif.

[6a] Received by the Secretary February 25, 1936

[7] "La Statique Graphique", Pt. III, by Maurice Levy.

nal work with respect to the external bending moment and the angular change has the same sign as that of the moment, that is,

$$\theta \text{ or } \tau = \frac{dW}{dM} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(31)$$

By virtue of its derivation Equation (31) involves the concept that the angular change occurs either in the same plane, or in a plane parallel to the actuating bending moment. Although the accuracy of Equation (2) is admitted, it is not obvious to the writer that the torsional moment, m_v, contributes anything to the value of θ, or that the bending moment, m_u, contributes anything to the value of τ. If these views are correct, there will be no partial differential coefficients, Equations (3) and (4) will be modified, and Equations (7) to (10), inclusive, will be superfluous.

The formula for the angular rotation about the normal to the tangent at any section of an arch rib in terms of an external bending moment is,

$$\theta = -\int \frac{M_b \, ds}{E \, I_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(32)$$

and that about such a tangent is,

$$\tau = +\int \frac{M_t \, ds}{G \, I_t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

in which M_b and M_t are, respectively, the external bending and torsional moments at the section; ds is a small piece of the arch axis; E and G, respectively, are the moduli of elasticity in tension; and, in shear, I_n and I_t are the rectangular moment of inertia about the normal and the polar moment of inertia about the tangent, respectively.

The limits of the definite integrals are the springing line and the distance from it to the section of the arch rib measured along its geometric axis. The sign convention is that of the author, θ and τ are independent variables along the rib and are proved to be so by Professor Levy[8] from simple geometric considerations.

Assume for the moment that the torsional moment, m_v, has some effect upon θ and that the bending moment, m_u, has some effect upon τ. Then, C_{1m} is obtained by partial differentiation of Equation (5) with respect to m_v, which amounts to finding the first derivative of cos ϕ, which is $-$ sin ϕ. Likewise, C_{1t}, the first derivative of sin ϕ, is $+$ cos ϕ, allowing M_w to retain its negative sign. Similarly, C_{2m} and C_{2t} are, respectively, $-$ sin ϕ and $-$ cos ϕ. When substituted in Equations (11) to (13), inclusive, these values will alter their form. Equation (16), being derived from Equation (14), reveals that $M_1 = -1$ always, whereas its evaluation from Equation (15) would also $= -1$. Equations (14) and (15) express the fact that θ_1 always equals τ_1, which is doubtful. Similar doubt exists as to M_2 from Equations (17) to (19) which would yield two different values of M_2.[8a]

[8] "La Statique Graphique", p. 183 *et seq.*

[8a] For corrections to the paper refer to last paragraph of this discussion.

A statement should be made as to the evaluation of the lateral shear in the rib. In 1918, Professor Levy[9] stated that, for an arch with symmetrical shape and loading, the torsion and shearing stresses are zero at the crown. He also stated that the total external shear at any section is equal to the algebraic sum of all the lateral forces between the crown and the section in question. This external shear results in internal shearing stress at the section, which must be combined properly with similar stresses due to the torsional moment and the vertical shear due to gravity loads.

The author closes the development of the first caption with a statement as to the limits of the integrations for Equations (16) to (19). It would assist those not so familiar with the method if this were done for the formulas preceding Equation (16).

Before closing the discussion of the subject under the caption, "Arch Ribs Without Bracing", the writer directs attention to the statements by the author that direct stress is induced by wind in the rib. Since the arch subjected to lateral forces acts as a beam which is subjected to bending, torsional shear, and ordinary beam shear, and has no curvature in the horizontal plane, the writer does not see how direct stress without a prerequisite arch thrust can occur.

Another statement of theory which must be questioned is the second paragraph of the author's "Conclusion." Professor Levy[10] shows that the action of gravity forces leaves the vertical neutral plane undeformed, whereas the lateral forces produce displacements normal to that plane, and "each of these kinds of forces produce the same displacements as if the other did not exist." Therefore, the lateral wind forces can have absolutely no effect on the forces in the plane of the arch rib.

Braced Arch Ribs.—The author's assumption that each rib is subjected to the same wind load happens to fit in nicely with the development of his bracing theory for two symmetrical arches, but what would he do if the bridge were on a curve with one arch longer and thicker than the other? If there were three or more ribs in the bridge unequally spaced, or of different dimensions, or both? The correct guess as to wind-load distribution between the ribs would be difficult. Professor Levy has shown[11] how to treat this problem with only the generally accepted assumptions.

By Equations (29) and (30)[8a], the values of R_1 and R_2 are approximately 3 150 lb and 2 018 lb, which corresponds with the writer's experience that the horizontal bending moment and the shear it causes in the brace are more serious than the torsional bending moment and its resultant shear. The combined bending and direct wind stress is equal to $\pm$ 528 lb per sq in., which indicates a large factor of safety for the 24 by 24-in. brace. However, the brace should always be analyzed for shear due to bending and for other column considerations. If the underlying assumptions are accurate, this method of analysis is an elegant one.

[9] "La Statique Graphique", Pt. III, p. 199, Paragraph *b*.
[10] *Loc. cit.*, Pt. III, p. 190, Lines 29 to 31.
[11] *Loc. cit.*, Pt. III, p. 211.

Professor Levy's Method.—Professor Levy has presented analyses of arches subjected to lateral loads for all shapes and degrees of fixation, but the writer will merely touch upon the symmetrical shape and loads to indicate the rapidity, and the ease of visualizing the structure during such analyses. Fig. 4 shows a pair of such ribs in plan, and the elevation of the rib shows all the work required for the determination of the bending moment and the

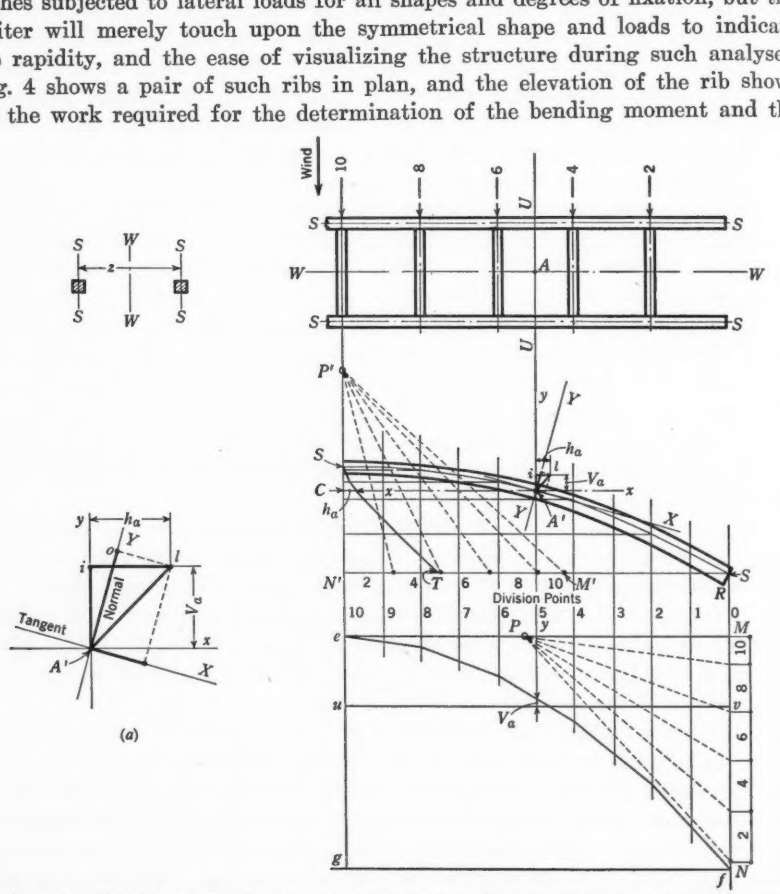


FIG. 4.—EVALUATION OF HORIZONTAL AND TORSIONAL BENDING MOMENTS, LEVY METHOD.

torsional moment at any section. The leeward rib receives its wind loads at Points 10, 8, 6, 4, and 2, and these loads are laid off in order, to yield the force polygon, *P–M–N*. For convenience, the pole distance, *P–M*, was taken as the sum of the loads, *M–N*, and the equilibrium polygon, *e–f*, was drawn through the Load Points 10 to 2. The closing line, *u–v*, of the arch considered as a fixed-ended beam is located by laying off from Line *g–f* the arithmetic mean of all the ordinates through Points 10 to 0 between the line, *g–f*, and the polygon, *e–f*, taken to the dimension scale of the layout. Then, the ordinate, v_a, between Lines *e–f* and *u–v*, multiplied by the pole distance, *P–M*, is the bending moment at Point 5 about the vertical plumb-line, *y–y*.

By drawing another force polygon for the same loads laid off on the half-chord, *N′–S*, and drawing an equilibrium polygon through horizontals drawn through the load points, 10, 8, 6, 4, and 2, the torsional moment about the

horizontal through Point 5 is equal to h_a times the pole distance for the force polygon. These two values correspond in direction to the values obtained by the author when he multiplies R_1 by one-half the brace length in the problem and does the same with R_2.

The construction in Fig. 4 permits evaluating the bending and torsional moments about Axis Y–Y at Point 5 and about the tangent itself from the corresponding moments about the vertical axis, y–y, and horizontal axis, x–x, through Point 5. Fig. 4(a) shows this construction, at Point 5 on the rib, enlarged. These new values are necessary, of course, for the analysis of the rib itself.

Most of the groundwork for this analysis having already been done in solving the problem of the gravity load, the solution of the arch for wind is rapid. The concepts of loads and force polygons are familiar to most engineers; relative magnitudes can be compared and it is seen that the point where the polygon, e–f, crosses the line, u–v, is the point of minimum bending. It is also a point of relatively small torsional moment.

Rational Method for Analyzing a Braced Arch System.—The author's development of formulas and his "Illustrative Example" show clearly what he means by the assumption that each rib is subjected to the same wind load; that is, the leeward rib of the system resists the same amount of wind as the windward rib. This assumption, made in order to arrive at a solution, penalizes the system too severely and is entirely unnecessary. Professor Levy has shown[12] that a pair of ribs may be considered as plate girder flanges with the same simplifying assumption utilized by D. B. Steinman, M. Am. Soc. C. E., for the stiffening trusses of suspension bridges.[13] In short, the moments of inertia of the ribs about their own gravity axes are neglected, and the moment of inertia of the pair equals twice the area of one rib, multiplied by one-half the square of the effective depth of the pair when both ribs are similar. Applying this concept to the polar moment of inertia corresponding to the torsional bending moment, the polar moment becomes a rectangular moment of inertia.

Use of these constants with the familiar formula for fiber stress in simple bending, utilizing the bending and torsional moments found for a specific rib section, will yield stresses fairly close to the actual. The only assumption made in the process has been held valid by those who discussed Mr. Steinman's classic paper. In this method, the pair of ribs is caused to withstand the wind force coming on only one rib (that is, the windward rib), with resultant economy.

If each brace of the type studied by the author were designed for the bending and direct stress occurring at the section at which it is framed to the rib, the size of the brace will not be excessive. For instance, if the arch in the author's problem were supplied with a brace, 24 by 24 in., at the crown, and his value for the bending moment, 92 785 ft-lb (which the writer checks closely for wind on one rib) is used, $f_b = \dfrac{6 \times 92\,785 \times 12}{24 \times 24 \times 24} = 485$ lb per sq in.

[12] "La Statique Graphique", p. 211 *et seq.*

[13] "A Generalized Deflection Theory for Suspension Bridges", *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 1223, Lines 10–12.

Assuming an average depth of rib of 4 ft and three braces in the entire rib: $f_c = \dfrac{2 \times 67.30 \times 4 \times 45}{3 \times 24 \times 24} = 24$ lb per sq in.

The total bending and direct stress equals 509 lb per sq in., which leaves ample margin for shear in the brace induced by bending. A study of the mechanics of a braced system of ribs, whether two or more ribs are involved, will show that distribution of the moment existing in the rib at any given section to the brace will depend upon the relative stiffness, $\dfrac{I}{L}$, of the brace, and the moment taken by the brace will be less than the moment in the rib. Esthetics, however, will not permit of a brace section commensurate with the stresses therein, so that the writer's severe but simple method used in connection with Professor Levy's method, will not result in excessively large braces.

The writer finds the bending moment at the crown to be 99 700 ft-lb; that in the rib at the brace, 44 700 ft-lb; and the torsional moment in the rib at the brace, 21 000 ft-lb. The values found by the author in the rib at the crown, were 92 785 ft-lb, and with corrected formulas the bending in the brace would be 47 250 ft-lb, with a torsional moment in the brace of 30 270 lb. The writer believes the author's values for the braces are too high.

Mr. Eremin has opened a discussion of the treatment of the simplest types of arch-rib bridges in which the ribs and the loading are symmetrical. When either of these items are unsymmetrical, the solution is not as simple as the author's conclusion indicates. It is to be hoped that still other methods of analyzing structures subjected to lateral loads will be developed, which are simpler, easier to grasp, and more rapid.

The following corrections to the paper are being made before final publication in *Transactions*: In Line 2 following Fig. 2, change τ to θ and θ to τ; in the line preceding Equation (20) change I_b to I_a; in Equations (20), (22), and (24), change τ to θ; in Equations (21), (23), and (25), change θ to τ; in Equations (20) and (24) change I_b to I_a; and, in Equations (23) and (25), change I_a to I_b.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SUCCESSIVE ELIMINATION OF UNKNOWNS IN THE SLOPE DEFLECTION METHOD

Discussion

BY ADOLPHUS MITCHELL, JUN. AM. SOC. C. E.

ADOLPHUS MITCHELL,[13] JUN. AM. SOC. C. E. (by letter).[13a]—The recent trend has been to overlook the advantages of an algebraic procedure of applying the slope-deflection equations in favor of arithmetical methods even when dealing with relatively simple frames. For this reason Professor Wilbur's ideas are especially interesting.

Unquestionably, algebraic solutions are preferable when analyzing frames, the design of which must be made at frequent intervals, whenever the equations can be handled with reasonably little effort. Among the structures composed of constant *I* members, the writer has found that triple culverts and six-span continuous beams yield without difficulty to the algebraic solution. Structures consisting of tapering members are more difficult to solve, but single-story bents with two columns and three-span continuous beams are easy enough. However, in the writer's opinion, a skeleton type of building frame, such as that described by the author, should, unhesitatingly, be solved by fixation factors[14], or by moment distribution.[15] If the slope-deflection method is applied, the use of Gauss' normal equations[16] will be found valuable as it affords many checks as the solution proceeds.

The importance of arranging the solution so as to produce the greatest degree of accuracy cannot be over-emphasized. Although in the schoolroom it is sufficient to gain the principles involved, in the design office, the quantitative results must be correct as well. In the treatment of continuous beams and frames of limited complexity it is best to consider all members loaded and

NOTE.—The paper by John B. Wilbur, Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1936, by Messrs. C. A. Willson, Paul Andersen, and R. W. Stewart.

[13] Structural Designer, State Highway Dept., Santa Fe, N. Mex.

[13a] Received by the Secretary February 6, 1936.

[14] "Structural Frameworks", by T. F. Hickerson, M. Am. Soc. C. E., Univ. of North Carolina Press., Chapel Hill, N. C.

[15] *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[16] "Secondary Stresses in Bridges", by Cecil von Abo, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 107; also, "Method of Least Squares", by Merriman, N. Y., John Wiley & Sons.

to solve the problem in such a manner that a symmetry of form in the equations is preserved throughout the analysis. One will soon find this an inestimable aid to accuracy.

Consider the problem of a continuous beam over five supports and with equal spans, as shown in Fig. 6. Let $A = -C_{1,2}$; $B = C_{2,1} - C_{3,4}$; $C = C_{3,2} - C_{3,4}$; $D = C_{4,3} - C_{4,5}$; and $E = +C_{5,4}$; and notice that for uniform loading over all spans the loading factors, B, C, and D, equal zero. Next, write the equation for each support, omitting EK for simplicity; thus:

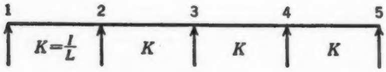


FIG. 6.

$$M_{1,2} = 0 = 2\,\theta_1 + \theta_2 + A$$

$$M_{2,1} + M_{2,3} = 0 = 4\,\theta_2 + \theta_3 + \theta_1 + B$$

$$M_{3,2} + M_{3,4} = 0 = 4\,\theta_3 + \theta_4 + \theta_2 + C$$

$$M_{4,3} + M_{4,5} = 0 = 4\,\theta_4 + \theta_5 + \theta_3 + D$$

and,

$$M_{5,4} = 0 = 2\,\theta_5 + \theta_4 + E$$

To eliminate the unknowns, begin simultantously at each end, as indicated by the arrows:

$$\theta_1 = -\frac{1}{2}\,\theta_2 - \frac{1}{2}\,A$$

$$\frac{7}{2}\,\theta_2 + \theta_3 + B - \frac{1}{2}\,A = 0; \text{ or } \theta_2 = -\frac{2}{7}\,\theta_3 - \frac{2}{7}\,B + \frac{1}{7}\,A$$

$$\frac{26}{7}\,\theta_3 + \theta_4 - \frac{2}{7}\,B + \frac{1}{7}\,A + C = 0;$$

$$\text{or, } \theta_3 = -\frac{7}{26}\,\theta_4 - \frac{7}{26}\,C + \frac{2}{26}\,B - \frac{1}{26}\,A \downarrow$$

$$\theta_3 = \frac{2}{26}\,\theta_3 + \frac{2}{26}\,D - \frac{1}{26}\,E - \frac{7}{26}\,C + \frac{2}{26}\,B - \frac{1}{26}\,A \longrightarrow \theta_3$$

$$= \frac{1}{24}\Big[-A + 2\,B - 7\,C + 2\,D - E\Big]$$

$$\uparrow \frac{7}{2}\,\theta_4 + \theta_3 + D - \frac{1}{2}\,E = 0; \text{ or } \theta_4 = -\frac{2}{7}\,\theta_3 - \frac{2}{7}\,D + \frac{1}{7}\,E$$

$$\theta_5 = -\frac{1}{2}\,\theta_4 - \frac{1}{2}\,E$$

Finally:

$$\theta_2 = \frac{1}{83}\,[13\,A - 26\,B + 7\,C - 2\,D + E]$$

$$\theta_4 = \frac{1}{84}\,[A - 2\,B + 7\,C - 26\,D + 13\,E]$$

$$\theta_1 = -\frac{1}{168}\,[97\,A - 26\,B + 7\,C - 2\,D + E]$$

and,

$$\theta_5 = -\frac{1}{168}\,[A - 2\,B + 7\,C - 26\,D + 97\,E]$$

Substituting in the fundamental equations.

$$M_{2,1} = 2\,\theta_2 + \theta_1 + C_{2,1} = \frac{1}{56}\,[-\,15\,A - 26\,B + 7\,C - 2\,D + E] + C_{2,1}$$

$$M_{3,2} = 2\,\theta_3 + \theta_2 + C_{3,2} = \frac{1}{56}\,[+\,4\,A - 8\,B - 28\,C + 8\,D - 4\,E] + C_{3,2}$$

and,

$$M_{4,5} = 2\,\theta_4 + \theta_5 - C_{4,5} = \frac{1}{56}\,[+\,A - 2\,B + 7\,C - 26\,D - 15\,E] - C_{4,5}$$

Notice the symmetry preserved in each step. The writer now wishes to justify his claim that "it is best to consider all members loaded." There are almost always limiting cases for which the solution of a problem can be found. In this problem it is the condition of uniform load over all spans, which may be used as a check. For this case, $A = -\frac{1}{12}\,w\,L^2$; $B = 0$; $C = 0$; $D = 0$; $E = +\frac{1}{12}\,w\,L^2$; and $M_{3,2} = \frac{1}{56}\left(\frac{1}{12}\,w\,L^2\right)(-4\ -4) + \frac{1}{12}\,w\,L^2 = \frac{2}{28}\,w\,L^2$, which value can be found in any handbook. Although this check does not necessarily indicate that every part of a solution is correct, failure to obtain it does reveal that a mistake has been made. The thoroughness of this method of checking depends upon the number of known cases available. If there is a case requiring the retention of each loading term, the check may be regarded as absolute.

If the reader would write out a solution carrying θ_1 as the unknown throughout the evaluation of the rotations as suggested by the author, he would more readily appreciate the foregoing procedure

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

REINFORCED CONCRETE MEMBERS UNDER DIRECT TENSION AND BENDING

Discussion

BY MESSRS. RALPH E. BYRNE, JR., CARL H. HEILBRON, JR., F. E. TURNEAURE, H. E. WARRINGTON, WILLIAM A. LARSEN, AND B. KOVEDIAEFF

RALPH E. BYRNE, JR.,[9] JUN. AM. SOC. C. E. (by letter).[9a]—The problem of combined tension and bending has been presented very clearly in this paper. However, when part of the section is in compression, the case of tension and bending is one part only of the more general problem of bending and direct stress, the case of compression and bending being the other part. Furthermore, by re-arranging the terms in the equations derived by the author, these equations may be obtained in forms which lend themselves more readily to solution by means of curves.

Consider the case in which part of the section is in compression, and the solution is related to the center of the reinforcement in the tension face, there being no reinforcement in the compression face. The author's notation and sign convention will be used with the following exceptions: N will be used to denote the resultant direct stress, either tension or compression, and will be considered positive when the direct stress is compression. For the case being discussed, eccentricities are measured from the center of the tension steel, and will be assumed positive when measured toward the compression face of the member; that is, in Fig. 4 of the paper, positive eccentricities are measured upward from the steel. It should be noted that the part of the section in compression is always on the positive side of the steel as defined by this sign convention.

Taking moments about the center of the tension steel,

$$N e' = \frac{1}{2} f_c\, k \left(1 - \frac{k}{3}\right) b\, d^2 \quad \text{.................} (39)$$

NOTE.—The paper by D. B. Gumensky, Assoc. M. Am. Soc. C. E., was published in December, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1936, by Messrs. A. W. Fischer, William E. Wilbur, and F. C. Snow.

[9] Structural Designer, Pacific Elec. Ry., Los Angeles, Calif.

[9a] Received by the Secretary February 7, 1936.

Equating the sum of the horizontal forces to zero,

$$-N + \frac{1}{2} f_c\, k\, b\, d - A_s f_s = 0 \quad \text{(40)}$$

Equation (40) differs from Equation (21) of the paper only in the sign of the direct stress; this difference is accounted for by the change in sign convention suggested by the writer. Equation (39) is identical with Equation (20). Since the right member of Equation (39) is always positive, it is evident that N and e' must always be of the same sign; that is, positive direct stress (compression) must always be accompanied by positive eccentricity, and negative direct stress (tension), by negative eccentricity. A third equation is obtained from the assumption of straight-line distribution of stress,

$$\frac{f_s}{n\, f_c} = \frac{1-k}{k} \quad \text{(41)}$$

Equations (39), (40), and (41), solved simultaneously, will yield the same expressions as those obtained by the author, except that all terms containing e' will be of opposite sign. However, other forms of these equations more readily lend themselves to solution by means of curves.

Problems of bending and direct stress which fall into the case being considered may be divided roughly into two types: (*a*) Those in which the eccentricity is small compared to the depth of the member; and (*b*) those in which the eccentricity is large compared to the depth of the member. For the first type, curves plotted using values of the quantity, $\frac{e'}{d}$, will prove advantageous —that is, curves similar to Fig. 5. The chief objection to these particular curves, however, is that a separate set of curves is required for each value of n. This objection may be overcome as follows: Rewrite Equation (23):

$$\frac{T\, n}{f_s\, b\, d} = p\, n - \frac{k^2}{2\,(1-k)} \quad \text{(42)}$$

Using Equation (42), a set of curves similar to those in Fig. 5 could be plotted, using values of the quantity, $p\, n$, as abscissas, values of k as ordinates, and drawing curves of the quantities, $\frac{e'}{d}$ and $\frac{T\, n}{f_s\, b\, d}$. This single set of curves would then cover any problem that falls into the case being considered, and by adopting a sign convention similar to that offered in this dis-

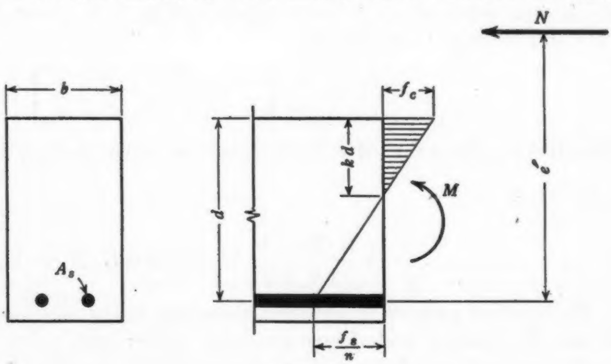


FIG. 6.

cussion, the curves could be extended to cover the entire range of both tension and compression. For the second type, that is, problems in which the eccentricity is large compared to the depth of the member (see Fig. 6), curves plotted using values of the quantity, $\frac{d}{e'}$, will prove more advantageous. In Fig. 6, the quantities used in plotting the curves in Fig. 7 are defined; thus, $M = N\,e'$; $f_c = C\,\frac{M}{b\,d^2}$; and $R = \frac{f_s}{n\,f_c} = \frac{1-k}{k}$. It is to be noted that N and e are both positive when the direct stress is compression, and negative when the direct stress is tension. The moment, M, is always positive. The curves in Fig. 7 are for cases of large eccentricity, and were plotted from Equation (44),

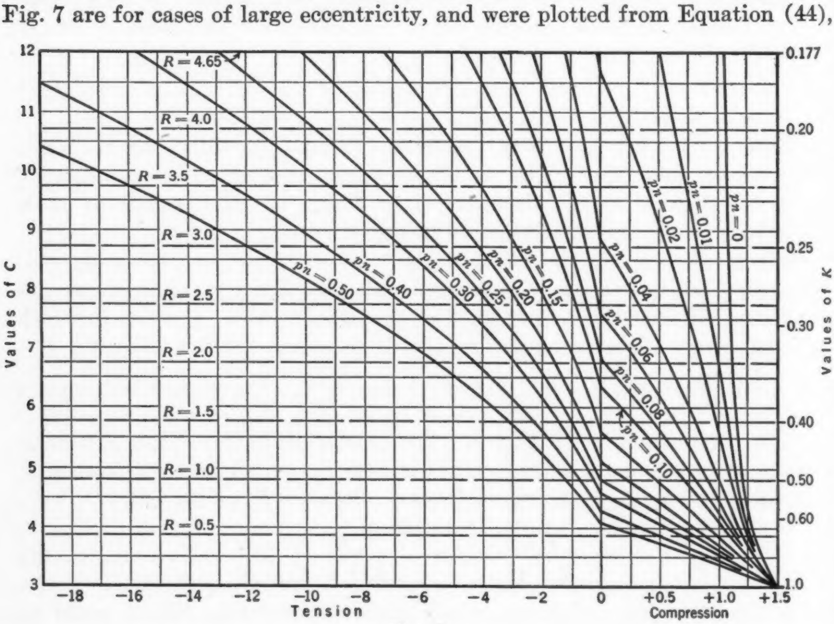


FIG. 7.

which was obtained as follows: Eliminating N b tween Equations (39) and (40) and letting $A_s = p\,b\,d$:

$$\frac{f_s}{f_c} = \frac{k}{2\,p}\left[1 - \left(1 - \frac{k}{3}\right)\frac{d}{e'}\right] \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Substituting the value of k from Equation (41) in Equation (43) and letting $R = \frac{f_s}{n\,f_c}$:

$$\frac{d}{e'} = \frac{3\,(R+1)}{3\,R+2}\,[1 - 2\,p\,n\,R\,(R+1)] \ldots\ldots\ldots\ldots (44)$$

Probably of greater importance than any of the cases treated by the author is that of bending and direct stress in which both sides of the member are reinforced, and in which part of the section is in compression. In this case,

again, the solution is general for both tension and compression. The case in which the direct stress is compression has already been amply treated[10], and need not be repeated here. Noting that direct tension must be accompanied by negative eccentricity, it is evident that all equations derived for the case of bending and compression can be applied directly to the case of bending and tension if the sign of every term which contains e is reversed; curves which cover the case of bending and tension are merely extensions of the curves which cover the case of bending and compression.

CARL H. HEILBRON, JR.,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—Useful and convenient charts are presented in this paper, for solving the usual type of problem of combined tension and bending in concrete members with reinforcement in a single layer. The writer believes that curves of the type of Fig. 5, rather than Fig. 3, will be of greater use, since a smaller number of diagrams will be required for the solution of all problems.

In using Fig. 5, it is rather difficult to follow some of the lines because of their curvature, and hard to determine the intersections accurately because of the sharp angles. To overcome these troubles the writer suggests the use of Fig. 8, which is fundamentally the same as Fig. 5, being based on the same equations. It is to be used in exactly the same manner. In addition to the change of arrangement, permitting straighter lines and better intersections, a further modification is the use of the functions, $n\ p$, $\frac{n\ T}{f_s\ b\ d}$, and $\frac{f_s}{n}$, instead of p, $\frac{T}{f_s\ b\ d}$, and f_s, respectively, which allows one diagram to serve for all values of n.

The use of this diagram is illustrated by the solution of Example 2 of the paper. The given conditions of the problem are: $M = 19\,230$ ft-lb.; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and, $d_c = 2$ in. As before, the value of the ratio, $\frac{e'}{d}$, is found to be 1.0. Taking an allowable tensile steel stress of 18 000 lb per sq in., one finds that $\frac{n\ T}{f_s\ b\ d} = 0.115$. Entering the left-hand part of Fig. 8, with $\frac{e'}{d} = 1.0$ and $\frac{n\ T}{f_s\ b\ d} = 0.115$, one reads, at the intersection of these two values, $n\ p = 0.247$, from which $p = \frac{0.247}{15} = 0.0165$ and $a_s = 0.0165 \times 10 \times 12 = 1.98$ sq in. Projecting horizontally from the intersection just found to the right-hand part of the chart, one finds, at the intersection with the line, $\frac{f_s}{n} = \frac{18\ 000}{15} = 1\,200$, the value, $f_c = 800$ lb per sq in.

[10] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. B. Maurer.

[11] Asst. Engr. (Design), Metropolitan Water Dist. of Southern California, Los Angeles, Calif.

[11a] Received by the Secretary February 8, 1936.

The value found for a_s (1.98 sq in.) is very close to 1.973 sq in., as given by exact mathematical solution of the equations involved. The value obtained by the author ($a_s = 1.92$ sq in.), although sufficiently close for ordinary design purposes, is in error by an amount indicating inaccuracy in his diagrams.

Many engineers will probably prefer to use separate diagrams for various values of n. A diagram for any given value of n can easily be prepared from Fig. 8 by writing in the values of p, $\frac{T}{f_s\, b\, d}$, and f_s, for the value of n selected, on each of the lines for $n\, p$, $\frac{n\, T}{f_s\, b\, d}$, and $\frac{f_s}{n}$, respectively. Others may prefer diagrams of the type of Fig. 3, which can be modified in the same way as Fig. 5 was re-arranged to give Fig. 8, with the same resultant advantages.

The writer wishes to call attention to another method of solving the usual type of problem involving combined bending and tension (or bending and compression) where there is reinforcement on one side of the section only. This method requires no new chart, but makes use of the familiar diagram (found in any work on reinforced concrete design) giving the relation between $\frac{M}{b\, d^2}$, p, f_c, and f_s for simple bending. In any case of simple bending, the moment, M, at a section is equal to the moment of all internal forces on one side of the section about any point, such as the point in the section at the reinforcement. Consider a problem in which M, b, d, and p are given; then by means of the diagram, f_s and f_c can be found readily. Now, let the conditions of the problem be changed; let b and d be the same, but let it be a problem in combined bending and tension wherein a value of T is given and the value of M_s, the moment taken about the tensile steel, is identical with the value of M in the previous problem. Since the moment, M_s, is taken about the tensile steel, the tension, T, must be considered as acting in the line of the steel. The addition of this tension, T, acting at this point, will result in no change in the values of f_s and f_c that occur in the first problem, providing that, at the same time, enough steel is added at the same point to take the tension, T, at the existing stress, f_s.

To solve the second problem, considering M, p, b, and d as given, with a_s required to be found, one would proceed as follows: Enter the diagram with $\frac{M}{b\, d^2}$ and find a point at which f_c and f_s are satisfactory. Read p on the diagram; then the steel required is $a_s = (b\ d) \times (p \text{ from the diagram}) + \frac{T}{f_s}$. This formula can be proved by mathematical manipulation, but the logic of the foregoing argument is simpler and equally conclusive.

As an illustration of this method the problem of Example 2 of the paper will be solved. The data are: $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; and $d_c = 2$ in.; and a value of f_s of 18 000 lb per sq in. is to be allowed. The moment, M_s, about the steel is 164 600 in-lb. The

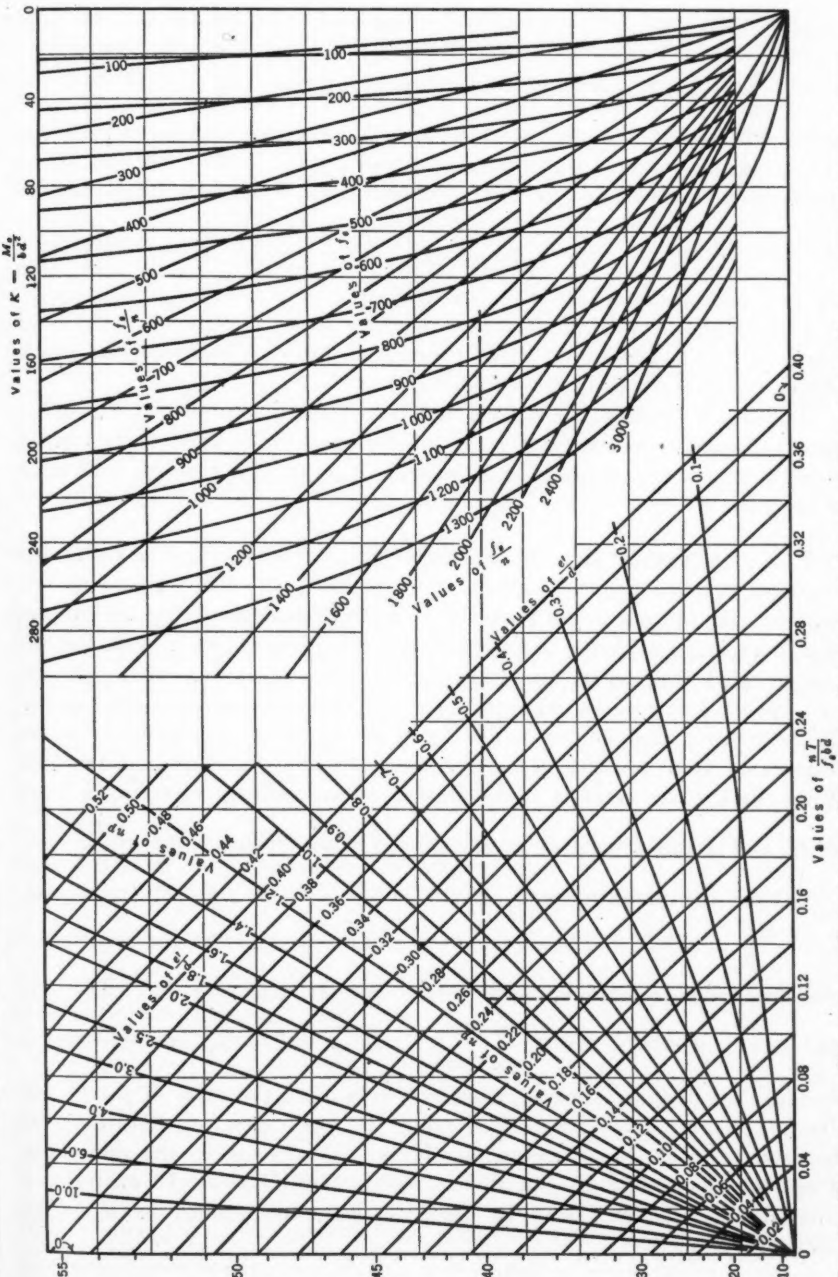


FIG. 8.—DIAGRAM FOR SOLUTION OF PROBLEMS IN REINFORCED CONCRETE, IN COMBINED BENDING AND TENSION WITH REINFORCEMENT IN A SINGLE LAYER. (MOMENT AND ECCENTRICITY ARE REFERRED TO THE CENTER OF GRAVITY OF THE TENSION STEEL).

value of $\frac{M_s}{b\,d}$ is $\frac{164\,600}{12 \times (10)^2} = 137.2$. Using a chart drawn for $n = 15$, enter with $\frac{M_s}{b\,d^2} = 137.2$, and find the intersection with the line, $f_s = 18\,000$ lb per sq in. The value of f_c here is 795 lb per sq in., agreeing with the value found in the author's example. The value of p read at this point is 0.00878. Then, the required area of steel is, by the aforementioned formula: The required $a_s = 12 \times 10 \times 0.00878 + \frac{16\,530}{18\,000} = 1.055 + 0.918 = 1.973$ sq in., which checks the values found by the other methods.

Still another method of solution which may appeal to some engineers is by means of a nomographic chart, which, of course, possesses the usual advantages and disadvantages of nomographs.

The choice between the various methods of solution presented, and other possible methods, will depend upon the frequency with which this type of problem is met, the accuracy of solution desired, and the personal preference of the user.

F. E. TURNEAURE,[12] HON. M. AM. SOC. C. E. (by letter).[12a]—The author has presented a very complete analysis of the problem of combined bending and tension. His diagrams are particularly useful for determining the stresses in a given beam subjected to known forces, but in designing a beam for certain specified working stresses a simpler method can be used. For this purpose the tension steel should be taken as the moment center, as done by the author in his second solution. The forces acting on the section may then be represented as a moment, M_1, and a direct tension, T, applied at the level of the steel. The beam is then designed as an ordinary rectangular beam for M_1 and the resulting steel area increased by an amount equal to $\frac{T}{f_s}$.

Taking the problem used by the author, $M_1 = 165\,300$ in-lb; and, $K = \frac{M_1}{b\,d^2} = 137.7$: For this value of K and for $f_s = 18\,000$, the usual tables or diagrams for rectangular beams give $p = 0.88\%$ and $f_c = 795$. The total steel area $= 0.0088 \times 120 + \frac{16\,530}{18\,000} = 1.97$ sq in. If the concrete stress is too high, the design can be adjusted by determining the value of p from the allowed concrete stress, thus increasing the steel area, or by increasing the value of d, as in any rectangular beam. Any change in the value of d, of course, will modify M_1, but this is readily re-calculated. In analyzing a given design, diagrams may be used, or the problem must be solved by "cut-and-try" methods.

[12] Cons. Engr.; Dean, Coll. of Mechanics and Eng., Univ. of Wisconsin, Madison, Wis.

[12a] Received by the Secretary February 14, 1936.

H. E. WARRINGTON,[13] M. AM. SOC. C. E. (by letter).[13a]—The problem treated in this paper was encountered in the design of the tunnel from the sewage treatment plant of the Los Angeles County (Calif.) Sanitation Districts to the Pacific Ocean. Diagrams such as those in the paper would have been of great service in that case. The same problem was treated quite fully by Professor Emil Mörsch in 1908.[14] The analysis, of course, is similar to that of the author, but the resulting diagrams are not. The requisite steel is not obtained directly as in this paper, hence, further computations are necessary.

In checking the results of the aforementioned studies with the author's diagrams, it happened, unfortunately, that $\frac{T}{f_s\, b\, d}$ and $\frac{e}{d}$ were far to the left upper corner, and their intersections were too indefinite, the curves being so nearly parallel. Mörsch's diagram does not suffer in this respect, although, in order to use it, a preliminary percentage of steel must be chosen. His graph then gives the value of k from that of $\frac{M}{T\, d_t}$, which is known. With k given, all stresses are determined at once. Therefore, the general formula,

$$e = \frac{M}{T}\,\frac{x^2(3\,d_t - 2\,x) + 6\,n\,p\,d\,(d - x)\,(2\,d - d_t)}{12\,n\,p\,d\,(d - x) - 6\,x^2} \quad \ldots\ldots(45)$$

was used, in which $x = k\, d_t$ and all other quantities are known. The cubic in x is best solved by trial approximation. Usually, two or three trials are all that are necessary. A check against numerical errors in the value of k is given by substituting the resulting foregoing data in the formula for,

$$M\text{ (in-lb)} = f_c\,\frac{b\,k\,d_t}{2}\left(\frac{d_t}{2} - \frac{k\,d_t}{3}\right) + f_s\,A_s\left(d - \frac{d_t}{2}\right) \quad \ldots\ldots(46)$$

For preliminary results, Mörsch treats the rectangular section as a beam, without reinforcement, by the well-known formula,

$$f_c = \frac{T}{b\,d_t} \pm \frac{6\,M}{b\,d_t^2} \quad \ldots\ldots(47)$$

thus finding the edge stresses. Then, he assumes all the tension to be taken by the steel, determining a quantity,

$$Z = \frac{b^2\,d_t^3\,f_c^2}{24\,M} \quad \ldots\ldots(48)$$

in which f_c is the tensile unit stress. This value of Z is the total steel stress, and the area required, of course, is $\frac{Z}{f_s}$. Trials of this method showed that the steel area thus found was a little larger than that given by the more

[13] Engr., Los Angeles County San. Dists., Los Angeles, Calif.

[13a] Received by the Secretary February 17, 1936.

[14] "Concrete Steel Construction", by Emil Mörsch; see, also, translation by E. P. Goodrich, M. Am. Soc. C. E., Engineering News Pub. Co., 1910.

accurate formulas. This method, by the way, is equally applicable to compression and bending. Possibly a diagram such as the valuable one for bending and compression, with single reinforcement, as presented[15] by F. E. Turneaure, Hon. M. Am. Soc. C. E., and Professor E. R. Maurer, could be made.

Mr. Gumensky is to be thanked for his valuable paper by all having to do with the problem.

William A. Larsen,[16] Esq. (by letter).[16a]—The topic of this paper is one that has been peculiarly avoided in the past. If it has been one's experience to design members of reinforced concrete structures for direct tension and bending, one will realize how little is recorded concerning it, but one must also realize that work of this nature has been constructed for some time. The reason for omitting discussions and charts on this topic in the American texts on concrete design is not fully understood by the writer. Simple bending and compression is a common problem and is covered fully in many books. Bending and tension is so closely related to it that it should offer no difficulty, as is shown by the author. This is also verified by other works.

It was pointed out by the author that the curves of Fig. 3 are less desirable than those of Fig. 5. The writer concurs and will direct the succeeding discussion primarily to the method presented, using the tensile steel as a reference for the eccentricity of the normal force.

The use of charts for the solution of equations similar to those encountered in this case not only expedites the work but also provides a graphical picture of the variables assisting the designer to visualize which of them has the most pronounced effect. It is best, therefore, to make the charts as simple and clear as possible. It is also desirable to keep them similar to other reinforced concrete diagrams in current use. The angle of intersection of the curves on a chart should approximate 90° if accuracy is desired. Based on these restrictions, the writer suggests a few changes which would make Fig. 5 more desirable.

The author finds the value of k by use of $\frac{T}{f_s\, b\, d}$ and $\frac{e}{d}$. The intersection of the lines representing these values is not definite in the upper part of the chart. As $\frac{T}{f_s\, b\, d}$ approaches zero and $\frac{e}{d}$ approaches infinity, the angle of intersection of these curves approaches zero. The points of intersection are thus obscured, especially if interpolation is necessary. It would be better to use other curves to obtain better intersections. With this purpose in mind, re-arrange Equation (20) in the form,

$$\frac{T\, e'}{b\, d^2} = \frac{f_c}{2}\, k \left(1 - \frac{k}{3}\right) = \frac{f_c}{2}\, k\, j \qquad (49)$$

[15] "Principles of Reinforced Concrete Construction", by F. E. Turneaure and E. R. Maurer, Fourth Edition, p. 434.

[16] With U. S. Bureau of Reclamation, Denver, Colo.

[16a] Received by the Secretary February 18, 1936.

This relationship is the same as has been used to plot the curves of the right-hand part of Fig. 5, if $\frac{M}{b\,d^2} = \frac{T\,e'}{b\,d^2}$. If $K = \frac{T\,e'}{b\,d^2}$ is used as the abscissa, k can be found by entering the chart at the bottom for a known K-value, trace upward to the intersection of either f_c, or f_s, and a value of k is thus defined. With the determined value of k, follow horizontally to the intersections with the desired $\frac{e'}{d}$ - line and thence down to the required p-value. This gives the same results as those obtained by the author, but without the use of the $\frac{T}{f_s\,b\,d}$ - curves. This also gives the same procedure as that used for charts on compression and bending[17]. The one operation gives, directly, the values for f_c and f_s in addition to the p-value. It is suggested, therefore, that the $\frac{T}{f_s\,b\,d}$ - curves be omitted, and $\frac{M}{b\,d^2}$ be changed to read $\frac{T\,e'}{b\,d^2}$. This will simplify the charts as well as their use.

The writer believes the curves as now shown, with values, $K = \frac{M}{b\,d^2}$, are misleading, thus emphasizing the need to change to $\frac{T\,e'}{b\,d^2}$. For instance, from Example 2, $K = \frac{M}{b\,d^2} = \frac{19\,230 \times 12}{12 \times 100} = 192.3$. Entering the chart at the lower right and tracing vertically to $f_s = 18\,000$, $f_c = 1\,000$ lb per sq in. (approximately, which is erroneous). Thus, using $\frac{T\,e'}{b\,d^2} = 134$, Point B is located, giving the correct values for f_c and f_s.

It is interesting to note that in 1918, E. Suenson[18] published a chart similar to Fig. 9(*b*) dealing with tension and bending, giving the same results as Fig. 5, but plotted to different co-ordinates. It is simple and easily applied. By adding a set of K, f_c, and f_s-curves (Fig. 9(*a*)), the chart can be used as was suggested for Fig. 5.

Fig. 9 is unique in that it was plotted as a bending and direct stress diagram, using $-N$ for tension. It can be used for either bending and tension, or bending and compression. For that reason it may be of greater value than Fig. 5. Its use is illustrated by Example 3.

Example 3.—In Example 1 of the paper, $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; and $d = 10$ in. The eccentricity relative to the geometric center $= \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163 = 14$ in. The eccentricity relative to the tension steel is equal to $e' = e - d + \frac{d_c}{2} = 14 - 10 + 6 = 10$ in.; and,

[17] "Concrete Designers Manual", by Hool and Whitney, pp. 179 and 180, McGraw-Hill Book Co., N. Y., 1926.

[18] "Jaernbeton-Teori og Praksis", by E. Suenson, 1918, P. E. Bluhmes Book Co., Copenhagen, Denmark.

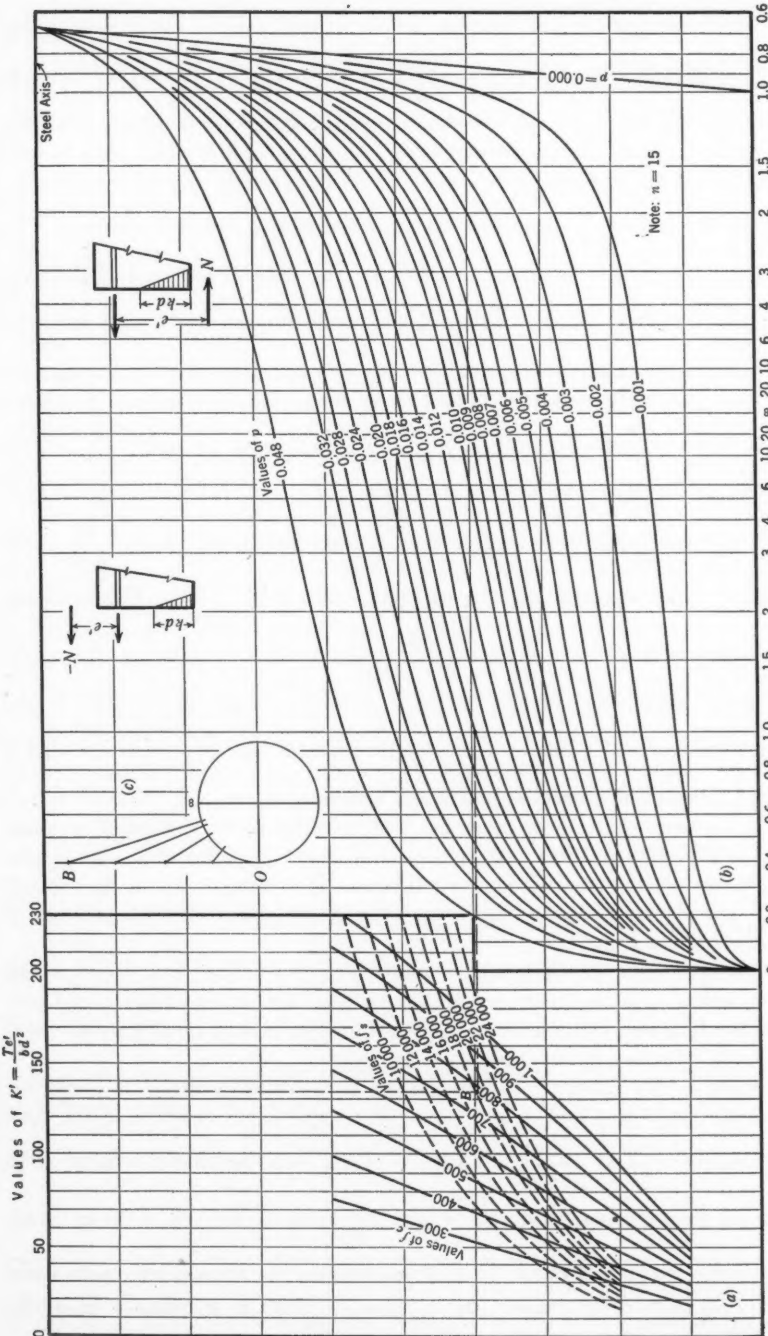

Values of $K' = \frac{Te'}{bd^2}$
Values of k
Values of $\frac{e'}{d}$
Values of p
Values of f_s
Values of f_c
Steel Axis
Note: $n = 15$
(a)
(b)
(c)

Fig. 9.

$\frac{e'}{d} = \frac{10}{10} = 1$. Furthermore, assume that $f_s = 18\,000$ lb per sq in., $\frac{T\,e'}{b\,d^2} = \frac{16\,530 \times 10}{12 \times 100} = 134$; then find the intersection of $\frac{T\,e'}{b\,d^2}$ and the f_s-curves, which gives $f_c = 795$ and $k = 0.395$ (Point B, Fig. 9(a)). Follow the horizontal line from Point B to the intersection of the vertical line, $\frac{e'}{d} = 1$, and p is found to be 0.016. These results are precisely the same as those found by the author in Examples 1 and 2.

One of the features of Fig. 9(b) is the plotting of the abscissas. The value of $\frac{e'}{d}$ ranging from 0 to ∞ necessitated a particular graph paper. A quadrant of a circle was used, the quarter circumference being assumed to represent a distance of 0 to ∞ (Fig. 9(c)). Points at equal distances were laid off on the tangent, $O\,B$, starting at Point O. Lines were drawn from these points to the center of the circle, the intercepts between these lines on the circumference being used as abscissa to plot the graph paper.

FIG. 9.

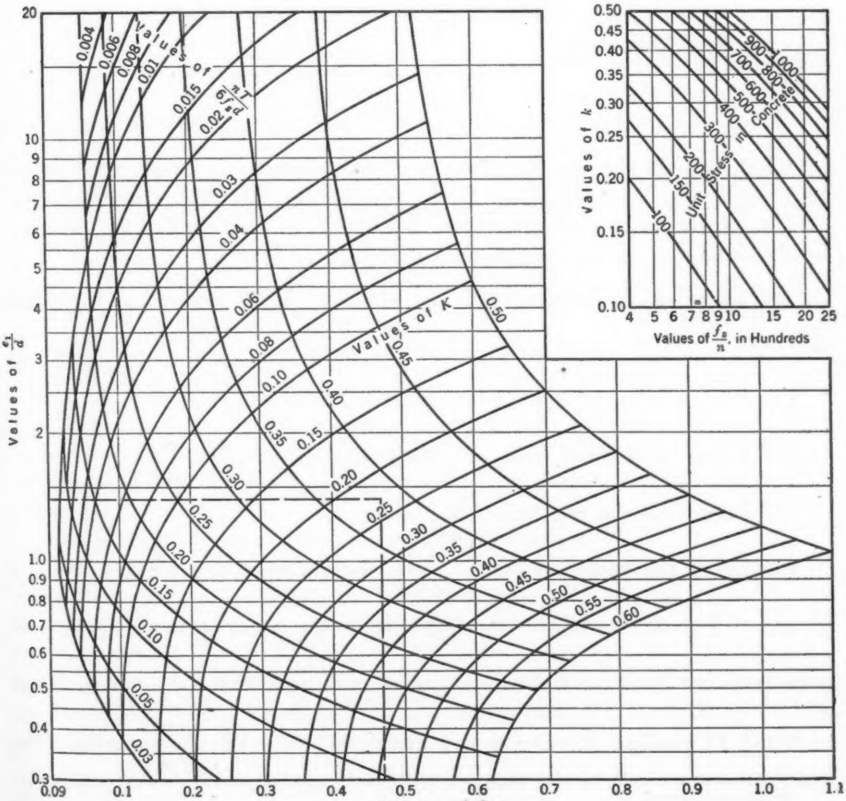


FIG. 10.

The necessity of using a different chart for each value of n is a serious disadvantage to diagrams of the type shown in Figs. 5 and 9(*a*). At present, there is a tendency to use n-values ranging from 6 to 15. To eliminate the need of so many charts J. A. Wineland, Jun. Am. Soc. C. E.,[19] in connection with his work at the Bureau of Reclamation, has prepared a diagram embodying n as a variable (see Fig. 10). Various co-ordinates were tried, using different combinations of variables until a diagram giving pronounced intersection of the curves was found. This diagram may be used for any value of n for tension and bending and, with a slight modification, for compression and bending. The diagram was plotted for members with a unit width of 12 in. Values of the ordinates are expressed in terms of,

$$\frac{e_1}{d} = \frac{k}{2}\left(\frac{2-k}{k+np}\right) + \frac{\left(1-\frac{k}{3}\right)\left(\frac{k^2}{1-k}\right)}{2\,np - \frac{k^2}{1-k}} \quad \ldots\ldots\ldots\ldots (50)$$

and, the abscissas are expressed by,

$$2\,np = \frac{k^2}{1-k} + \frac{n\,T}{6\,f_s\,d} \quad \ldots\ldots\ldots\ldots (51)$$

Furthermore, in Fig. 10(*b*),

$$f_c = \frac{f_s\,k}{n\,(1-k)} \quad \ldots\ldots\ldots\ldots (52)$$

in which, in addition to the notation of the paper, e_1 = eccentricity of the resultant force, T, with respect to the geometrical axis of the stressed areas.

The procedure in solving problems by Fig. 10 is very similar to that given by the author concerning the curves of Fig. 3:

(1) Knowing the moment, M, and the tension, T, determine the value, $e_1 = \frac{M}{T}$.

(2) With a trial value of d determine $\frac{e_1}{d}$.

(3) Using the desired value of n determine the value of $\frac{n\,T}{6 f_s\,d}$.

(4) Determine the relation of $\frac{f_s}{n}$.

(5) Using the values of $\frac{e_1}{d}$ and $\frac{n\,T}{6 f_s d}$, determine values for $2\ p\ n$ and k by use of Fig. 10(*a*). Enter the left side of the chart at the $\frac{e_1}{d}$-value and trace horizontally to the right to the intersection with the $\frac{n\,T}{6 f_s d}$-curve; k may be found by reading the curves practically 90° with the $\frac{n\,T}{6 f_s d}$-curves. The

[19] Not published.

value of 2 pn is found at the top of the chart directly above the point just located.

(6) With the value of k and $\frac{f_s}{n}$, Fig. 10(b) is used to find f_c.

(7) The necessary steel area is found by multiplying 2 pn by $\frac{b\,d}{2\,n}$.

It is seen that it is as easy to use Fig. 10 as Fig. 5 and only the one chart is necessary for all values. This compact diagram should appeal to designers who are concerned with various values of n.

The solution of a problem by the use of Fig. 10, with the moment as given in Eample 1, will give slightly different results from those found by the use of Fig. 3 or Fig. 5, because of the assumptions used in deriving the formulas. The author assumed the working lines to go through the geometrical axis of the gross concrete area. The curves of Fig. 10 were made feasible by assuming the working lines to go through the center of gravity or the geometrical axis of the stressed areas of the reinforced concrete member. This means that the value of the moment, M, will vary slightly, depending upon the working lines used. The eccentricity, $e_1 = \frac{M}{T}$, should be referred to the same working lines to which the moment is referred. Usually, however, sufficiently accurate results may be obtained by using the center line of the member as the working line, thus referring e_1 to the center line of the member.

Example 4.—The solution of the problem in Example 1, by use of Fig. 10 will give results comparable to those already obtained. For example: $M = 19\,230$ ft-lb; $T = 16\,530$ lb; $n = 15$; $d_t = 12$ in.; $d = 10$ in.; $b = 12$ in.; and $f_s = 18\,000$. Finally, $e_1 = \frac{M}{T} = \frac{19\,230}{16\,530} = 1.163$ ft $= 14$ in.; $\frac{e_1}{d} = \frac{14}{10} = 1.4$; $\frac{n\,T}{6 f_s\,d} = \frac{15 \times 16\,530}{6 \times 18\,000 \times 10} = 0.23$; and, $\frac{f_s}{n} = \frac{18\,000}{15} = 1\,200$. From Fig. 10($a$), 2 $pn = 0.47$, or $p = 0.0157$; and, $k = 0.385$. From Fig. 10(b), $f_c = 750$ lb per sq in.

If the moment, M, is referred to the proper working lines for use in Fig. 10, exactly the same results as those obtained in the other examples will be found.

The geometrical axis of the stressed area is given by:

$$y = \frac{k\,d\left(1 - \frac{k}{2}\right)}{k + n\,p} \qquad (53)$$

in which $y =$ the distance to the geometrical axis of the stressed areas from the steel axis. From Example 2: $k = 0.395$; $p = 0.016$; $n = 15$; $y = 5$ in., or 0.0833 ft above the center line; and,

$$M_1 = M + T\,(0.0833) \qquad (54)$$

in which M_1 = the moment referred to the geometrical axis of the stressed areas. In other words, $M_1 = 19\,230 + 16\,530 \times 0.0833 = 20\,608$ ft-lb. Finally $e_1 = \frac{M'}{T} = \frac{20\,608}{16\,530} = 1.25$ ft, or 15 in.; $\frac{e_1}{d} = \frac{15}{10} = 1.5$; and by the same procedure as in Example 3, $p = 0.016$ and $f_c = 795$ lb per sq in.

Conclusion.—The preference of the individual designer for the use of different diagrams and charts, the problems at hand, and the general designing practice will determine which type of diagrams are to be used. It is hoped that this discussion has added constructive information to that already given by the author to clarify this topic, which has been so carefully avoided in concrete design texts.

There is one more important question that should be considered carefully although it has not thus far been mentioned: What are the bond stresses and how may they be calculated? After the designer has conducted a careful analysis and has determined the required steel area by any method similar to those presented, it is necessary that he obtain the correct bond values. The familiar, $u = \frac{V}{b\,j\,d}$, will not satisfy the necessary requirements for bending and direct stress. The direct pull, T, tends to increase the tensile force in the steel over that caused by pure bending.

The change in the total tension per unit length of bar is equal to the bond stress. If a curve is drawn representing the total tension in the steel at any point along the length of the bar, the bond may be determined by the slope of the tangent to that curve. The unit bond stress, being equal to the bond divided by the peripheral area, may then be found. This is an accurate and easily remembered method.

B. Kovediaeff,[20] Esq. (by letter).[20a]—The subject of direct tension combined with bending in reinforced concrete members has been discussed in the more modern textbooks on reinforced concrete. However, the Engineering Profession, and especially engineers interested in structural work, will more than welcome a consistent discussion covering such an important subject.

In the "Introduction", the author states that the paper is an attempt to analyze the stress distribution in reinforced concrete members under direct tension and bending, and yet his first assumption is that of a "straight-line distribution of stress." There seems to be some inconsistency between these two statements. Obviously, if the author is attempting to analyze the stress distribution in the section, he should continue to do so instead of assuming it.

Case I, in which the entire section is under tension, has been very well treated by Messrs. Taylor, Thompson, and Smulski[2] and, therefore, no attempt need be made to carry the discussion further, since it deals with the reinforcement of both faces of the section, which is obviously not the nature of the problem undertaken by the author.

---

[20] With U. S. Engr. Office, Los Angeles, Calif.

[20a] Received by the Secretary February 24, 1936.

[2] "Concrete, Plain and Reinforced", by F. W. Taylor, S. E. Thompson, and Edward Smulski, Vol. 1, Fourth Edition, N. Y., John Wiley & Sons, Inc., 1925, p. 189.

The first true attempt at an analysis is begun by the introduction of Equation (8). Except for a change of sign and slightly different notation, the same equation is presented in one of the standard handbooks in current use.[21] Furthermore, in Equation (12), the author introduces the term, d_t, and, later, d_c, primarily for the purpose of simplifying the matter to follow; but these terms led the author into difficulties which could be overcome only by means of the introduction of a great number of diagrams. As an example, each diagram is dependent upon n and upon $a = \frac{d_c}{d}$. Using four values for n (8, 10, 12, and 15), and at least five values for a (0.1, 0.15, 0.20, 0.25, and 0.30), the total number of diagrams required to solve problems falling within this scope is twenty, which can scarcely be termed simple. Any value of a within the 0.05 interval would have to be interpolated, which would further complicate the problem.

It is apparent that the value, d_c (depth of cover on the steel), as shown in Fig. 2, has no direct bearing on the solution of the problem, because no matter how large d_c becomes, the distribution of stress remains unchanged. For this reason all formulas used for the determination of steel reinforcement in a single face disregard this value of depth of cover.

It appears that the author finally realized the inconvenience of the numerous curves necessary for the solution of these problems and decided to change the lever arm of the force-producing moments, by introducing Equation (22). Taking Equation (23) and solving for p, the expression for bending and tension is expressed as:

$$p = \frac{k^2}{2\,n\,(1 - k)} + \frac{T}{f_s\,b\,d} \quad \text{.................... (55)}$$

An identical equation can be written for bending and compression by changing the sign of the second term to minus; thus:

$$p = \frac{k^2}{2\,n\,(1 - k)} - \frac{C}{f_s\,b\,d} \quad \text{.................... (56)}$$

To obtain the expression for p required for the pure bending moment, the second term is omitted; thus,

$$p = \frac{k^2}{2\,n\,(1 - k)} \quad \text{........................ (57)}$$

Equations (55), (56), and (57) give three very simple expressions for steel percentage in single-face reinforcement. Determine $K = \frac{M}{b\,d^2}$, and from a set of diagrams of pure bending moments[22] (M being the transformed

[21] Concrete Engineer's Handbook, by Hool and Johnson, McGraw-Hill Book Co., 1918, p. 403.

[22] "Principles of Reinforced Concrete Construction", by F. E. Turneaure, Hon. M. Am. Soc. C. E., and E. R. Maurer, Fourth Edition, John Wiley & Sons, Inc., N. Y., 1932, pp. 414–415.

moment taken about the steel), read the value for p for pure bending as expressed by Equation (57). By one simple computation a value for $\frac{T}{f_s\, b\, d}$ is determined. Adding or subtracting this term to, or from, the value of p as found by Equation (57) will give the value of p for bending moment combined with tension or for bending moment combined with compression, respectively. Therefore, for the determination of the value of p in all three cases, the only diagrams required are those for the solution of pure bending. On the other hand, in the author's discussion, four diagrams for each case are required, or twelve in all.

This brief discussion points out the futility of a set of curves such as those suggested in the paper because of the simplicity of the relation existing between the forces, moments, and the steel ratio.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONSERVATION OF WATER

PROGRESS REPORT OF THE COMMITTEE OF THE IRRIGATION DIVISION

Discussion

By Messrs. Donald M. Baker, and Dean C. Muckel

Donald M. Baker,[5] M. Am. Soc. C. E. (by letter).[5a]—The work of the Committee in bringing together agencies involved in water conservation has been very constructive. Published proceedings of the two Conferences held in 1930 and 1935 illustrate the value of concentrating upon the problem of conservation the efforts of specialists with different backgrounds and methods of approach. Any subject as broad as the conservation of water has phases beyond the scope of specialists in any one field of activity and should be attacked by groups working on various fronts under co-ordinated direction.

The writer is in accord with most of the statements made in the Committee's report, but feels that one of them (see heading, "Evaporation") requires comment:

> "Sufficient direct observations are now available to enable evaporation to be estimated closely enough for most water supply purposes. Except for reservoirs of shallow depth or long periods of carry-over storage, evaporation is not a major factor in water supply development. Present information on evaporation from water surfaces is now generally adequate for nearly all engineering requirements."

It is felt that this pronouncement over-states the case. Probably present information on evaporation from water surfaces might be deemed adequate when one considers the quality of other basic hydrologic data available in making water supply studies, particularly the early records of stream flow and precipitation, and the few stations existing in former years; but it must be remembered that the average quality of these data is rapidly improving and that more and more information, of better and better quality, is becom-

Note.—The Progress Report of the Committee of the Irrigation Division on the Conservation of Water, was presented at the meeting of the Irrigation Division at Los Angeles, Calif., July 4, 1935, and published in December, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[5] Cons. Engr., Los Angeles, Calif.

[5a] Received by the Secretary February 13, 1936.

ing available as time goes on. It is hoped that a time is approaching when such data may be considered greatly superior in quality to those of existing evaporation data.

Evaporation becomes a more and more important factor in water supply development in the degree that it exceeds annual precipitation. Moreover, there are large areas of the United States in which this condition occurs, and it is usually in such areas, which have wide extremes in annual run-off, that hold-over storage is required. The importance of accurate knowledge of evaporation in connection with hold-over storage lies in the fact that use of erroneous values for evaporation in studies of such storage introduces a cumulative error which may cause appreciable differences in estimates of available water supply.

Furthermore, the writer does not agree that there are sufficient direct observations of evaporation. The statement made in the report might be taken as correct if one admits that an accuracy of ± 10% to 20% is all that is needed when estimating evaporation, but such accuracy is out of line with that of much of the present basic hydrologic data. The multitude of formulas proposed for computing evaporation are evidence that further information is needed to determine it at places where direct observations do not exist.

Furthermore, the relationship of pan evaporation to that from large open water surfaces has not been conclusively established under a variety of climatological conditions. Experiments by the late Reuben Benjamin Sleight, Assoc. M. Am. Soc. C. E.[6], at Denver, Colo., and Carl Rohwer, Assoc. M. Am. Soc. C. E.[7], at Fort Collins, Colo., appear to establish this relationship at those localities, but work at present under way in California indicates that a different relationship exists there, due probably to greater length of open season. Even if coefficients were established at various locations, that could be applied to translate evaporation from various sizes and shapes of pans to that from a pan 12 ft in diameter or a reservoir 85 ft in diameter, much more work is necessary to translate pan evaporation to that from open water surfaces of large area.

It is probably true that coefficients between small and large pans or tanks have been fairly well developed under Rocky Mountain conditions, and that a 12-ft pan of an 85-ft reservoir would be representative of a small area of open water surface at a given location upon a large reservoir with the water surface measured in square miles; but it is contended that the rate of evaporation at various points on the surface of a large reservoir must vary materially, due to differing climatological conditions, particularly on the windward and leeward sides of such reservoir. Such studies of evaporation from large open water surfaces as have been made to date, either by the use of pans, or by measuring the inflow and the outflow, have been inconclusive, being either of short duration or involving losses which occur from vegetation bordering the lake or reservoir. The results are affected by precipitation records taken

[6] "Evaporation from the Surfaces of Water and River-Bed Materials", by R. B. Sleight, *Journal of Agricultural Research*, Vol. 10, p. 209.

[7] "Evaporation from Free Water Surfaces", by Carl Rohwer, *Technical Bulletin No. 271*, U. S. Dept. of Agriculture.

at a distance from the site, lack of adequate accuracy in measuring inflow and outflow, etc. Much further work should be done along this line under varying climatic conditions.

If the statement quoted from the report is taken to mean that there is a sufficiency of scattered records of observations and of more or less individual attempts at research in evaporation, and that from now on organized and co-ordinated attack should be made upon the problem, the writer agrees entirely. Such a meaning might be inferred by the resolution passed by the 1935 Conference. However, such a program should include a large number of individual observations in order to obtain a spread of data under varying conditions.

Hearty accordance is had with the recommendations of the Committee with respect to the definition of the fields covered by various agencies engaged in water conservation and control and as to co-ordination of existing programs to prevent overlapping, duplication, and conflict. The foregoing activities fall in the following fields: (1) Collection of basic hydrologic data; (2) research in hydrology; (3) preparation of broad plans for water conservation and control; (4) construction of projects for water conservation and control; and (5) operation of projects for water conservation and control.

Each of the foregoing types of activity usually requires a different type of organization and personnel, although one organization may well, and often does, carry on two or more types of activity. Data may be collected and research studies may be made concurrently. Too often the making of general plans precedes the adequate collection of data, and research.

At present there are at least thirty-three Federal agencies engaged in the collection of basic hydrologic data, and, in addition, many State, local, and private agencies. Many of these agencies are engaged in research work, and also in other of the aforementioned fields. Some of these agencies collect data for other Federal agencies and transmit them for compilation and publication; other agencies publish the results of their collection or research activities themselves, immediately or after considerable lapse of time; whereas still others have no publication medium—at least of general availability.

The principal aim in the collection of hydrologic data, and in research work, is to make available material to the users of such data, who may be the agency collecting it or carrying on the research, or who may (as is usually the case) be some other public or private agency with activities more or less dependent upon such data or the results of research. Under present conditions the highest usefulness of results in the collection of data or in research work is far from being achieved, due to scattered and unco-ordinated activities.

Progress is being made in standardizing the collection, compilation, and publication of hydrologic data. The National Resources Committee has recently published a report[8] on the subject and its recommendations are being well received, but much additional work is necessary in co-ordination and

[8] Rept. of Special Advisory Committee on Standards and Specifications for Hydrologic Data, November 5, 1935.

in laying down a well-rounded program of collection and research. Such co-ordination should follow the lines of preventing overlap and duplication of activities, and the program should look toward the encouragement of more work along lines where inadequacies now exist, and toward making results of collection and research activities more easily available to users.

A word of caution in regard to planning activities is not amiss at this time. Too often planning enthusiasts become promoters and projects are encouraged before the time for them is ripe. All plans should be accompanied by a program for putting them into effect. The major purpose of plans is to insure the optimum development of water resources, and to prevent conflicts in development programs which will result in economic wastes. The purpose of the program is to time developments so that they come about when needed, and not before or after.

DEAN C. MUCKEL,[9] JUN. AM. SOC. C. E. (by letter).[9a]—This report is an excellent effort to summarize briefly some of the many problems confronting the engineer connected with water conservation projects in the West. The writer is particularly interested in that part of the report dealing with water-spreading.

As a means of conserving stream flow, water-spreading has taken an important position in the water program of that part of Southern California known as the South Coastal Basin, where approximately 90% of the water developed locally is pumped from underground reservoirs. A *Bulletin*[10] published in 1934 lists thirty-seven rather distinct ground-water basins with a total surface area of about 840 000 acres. It is estimated that every foot of average rise or fall of the water-table represents a change of about 70 000 acre-ft in the quantity of the underground water.. These subterreanean reservoirs, filled with porous alluvial material, tend to regulate the water supply derived from erratic precipitations by accumulating and storing the water of wet years for use in dry periods. They are replenished by rain falling directly on the valley floors, seepage from streams traversing the valleys, flood waters discharging over alluvial fans, and return water from irrigation. The natural methods of replenishment have been curtailed considerably with the development of the country. Means have been provided to collect run-off from paved streets and catchments from building roofs into storm sewers or into natural channels leading directly into the ocean. Flood channels which previously meandered across the valleys have been narrowed and confined to the smallest possible areas. These curtailments, together with an increased demand for water resulting from additional lands being brought under cultivation, and an ever-increasing population, have resulted in a general lowering of the water-table throughout the area.

In order to augment the natural methods of replenishment by artificial means, water-spreading grounds have been developed to encourage the perco-

[9] Jun. Engr., Div. of Irrig., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Pomona, Calif.

[9a] Received by the Secretary March 5, 1936.

[10] "Geology and Ground Water Storage Capacity of Valley Fill", *Bulletin No. 45*, California State Dept. of Public Works, South Coastal Basin Investigation.

lation of surface stream flow. Lands ideal for spreading purposes are found immediately below the mouths of the canyons where the streams leave the mountains. Large porous débris cones have been formed by previous floods rushing down the steep slopes in the mountains and carrying rocks, boulders, and débris which dropped as the velocity decreased when the stream reached the flatter slopes of the coastal plain. The surface areas of these cones vary from a few acres to more than 5 000 acres, depending on the size of the stream. In most cases, these areas would be wasted if they were not utilized for spreading, the rocks and boulders being of sufficient size to make the land unfit for agricultural use.

During the last four years (1932–1936) there has been considerable development of spreading areas in Southern California through the aid of Federal and local relief agencies, and there is now at least some form of spreading system on nearly every stream in the South Coastal Basin. The largest system (but not the most costly) is on the Upper Santa Ana River débris cone where the Water Conservation Association has set aside nearly 3 000 acres for spreading purposes and of which a very large portion can be wetted by the existing spreading works.

The primary purpose of spreading has been the conservation of water, but as the Committee reported "there has been some tendency to regard water-spreading as a method of flood control. This is erroneous as the capacity of spreading works can not be made sufficiently large to control major floods." Whether this tendency is entirely erroneous is questionable. Several of the larger spreading systems have been extended over many acres of land and by means of dikes and wire and rock contour dams the water is impounded in numerous surface basins. Neglecting percolation entirely, many spreading grounds are capable of storing, temporarily, several hundred acre-feet of water on the surface. Although they may not be sufficient to control major flood flows entirely, the spreading grounds have capacities sufficient to warrant some consideration in the flood-control program of the streams.

Unlike a surface reservoir, the actual capacity of a spreading system is not a constant or definitely known quantity because of the variation in percolation rates. Since the winter of 1929–30, the Division of Irrigation, Bureau of Agricultural Engineering, U. S. Department of Agriculture, has conducted investigations, under the direct leadership of A. T. Mitchelson, Senior Irrigation Engineer, to determine the most efficient methods of spreading and the various factors influencing the rate of percolation on water-spreading areas. These investigations were made on experimental plots under complete control and on areas where spreading is done on a large scale and under practical conditions. The tests show that the daily rate of percolation may vary considerably throughout a spreading period, although the seasonal or long-time averages for the several methods of spreading were found to be consistent. Daily rates on the same experimental plot were found to vary from 2.12 to 9.58 acre-ft per acre during the same period of spreading; in other words, the capacity of a spreading system might be approximately four times as great on one day as on another.

The report of the Committee states that 'there has also been a tendency to encroach on the natural flood channels by actual construction along their course or by failing to keep the channels clear of vegetation." This tendency is certainly hazardous. Regardless of whether or not a spreading ground exists on the shores of the stream, the flood channels should be designed and maintained with as much care as the spillway of a detention dam. The diversion of large flood flows is complicated because of the unstable condition of the stream beds across the débris cones. These cones are still in the process of formation, and each storm brings down varying quantities of silt, sand, gravel, and boulders. Diversion of all or of part of the flood water is thus made difficult because of the débris deposited around the head-works and in the diversion canals. On some cones the diversion of flood flows is further complicated by the fact that there is no well-confined flood channel and the introduction of a diversion structure of sufficient size to control major floods, by raising the elevation of the stream bed, may tend to throw the stream out of its natural channel.

In order to spread flood water, some provision must be made to handle the silt which is always present in such flows and is highly detrimental to percolation areas. Two methods have been devised for this purpose. On some spreading systems, settling basins have been constructed, through which the water must pass before being distributed to the spreading area proper. These basins may be equipped with gates leading back into the stream channel so that the accumulated silt may be flushed out periodically, or, in basins not equipped with gates, the silt must be removed by scrapers or other tools to maintain their usefulness.

Another system is the ditch method of spreading. The ditches are laid out on slopes sufficient to maintain a carrying velocity throughout the system so as to prevent deposition of silt on the porous ditch bottoms. At the lower end of the spreading area a collection ditch is used to divert the excessively silt-laden water back into the natural stream channel.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TALL BUILDING FRAMES STUDIED BY MEANS OF MECHANICAL MODELS

Discussion

BY MESSRS. L. C. MAUGH, L. J. MENSCH, AND GILBERT MORRISON

L. C. MAUGH,[4] ASSOC. M. AM. SOC. C. E. (by letter).[4a]—The use of small models in the study of complicated structural frames is undoubtedly a valuable adjunct to the various algebraic methods that are in current use. The results that are recorded must be considered as applying more to the conditions that exist in the usual hypothetical frame than in the actual tall building. From this viewpoint, the writer has made a comparison of the results that were obtained by the authors with those obtained by an approximate method that will be explained briefly in order that the comparison may be more complete.

In most regular building frames in which wind stresses are of any importance, the columns will be more rigid than the girders and will not vary greatly in stiffness from exterior to interior. For this condition, each bent can be considered as composed of a series of vertical Vierendeel trusses in which the columns are the chord members; the exterior columns act with only one truss whereas the interior columns are a part of two trusses. For Vierendeel trusses in which the chord members are approximately of equal rigidity, the writer has shown that the panel is the primary unit of the structure and can be used to provide a convenient method of solution by successive approximations.[5] The importance of the panel as a structural unit was also shown by Professor R. V. Southwell in his studies of stresses in rigid airships.[6]

NOTE.—The paper by Francis P. Witmer, M. Am. Soc. C. E., and Harry H. Bonner, Esq., was published in January, 1936, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] Asst. Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[4a] Received by the Secretary February 14, 1936.

[5] "Analysis of Vierendeel Trusses by Successive Approximations", by L. C. Maugh, Publications of the International Assoc. of Bridge and Structural Eng., Vol. 3; also, "A Rapid and Concise Method of Analyzing Rigid Viaduct Bents", *Engineering News-Record*, March 14, 1935.

[6] "On the Calculation of Stresses in the Hulls of Rigid Airships", by R. V. Southwell, R and M No. 1057, Aeronautical Research Comm. of Great Britain.

To illustrate the foregoing, let Fig. 10(*a*) represent a typical building frame of three bays. Panel *abed* will then be a part of the vertical truss, *T*–1, and Panel *bcfe* will be part of Truss *T*–2. In Fig. 10(*b*) Panel *abed* is considered as a rigid frame that is acted upon by a shearing force, V_o, and is restrained by the forces, $\frac{V_o h}{L_o}$, at Joints *d* and *e*. The horizontal displacement of Joints *a* and *b* with respect to Joints *d* and *e* will be equal to,

$$\Delta = \frac{V_o h^2}{24 E} \left(\frac{1 + n_o}{K}\right) \qquad (1)$$

in which n_o equals the ratio of the rigidity of the columns to that of the girder, $\left(n_o = \frac{K_c}{K_o}\right)$, and should be greater than unity to provide the truss action shown; Panel *bcfe* is similarly shown in Fig. 10(*c*) and in the same manner,

$$\Delta = \frac{V_i h^2}{24 E} \left(\frac{1 + n_i}{K}\right) \qquad (2)$$

in which V_i equals the shear taken by the panel and n_i equals the ratio of the rigidity of the columns to that of the girders $\left(n_i = \frac{K_c}{K_i}\right)$.

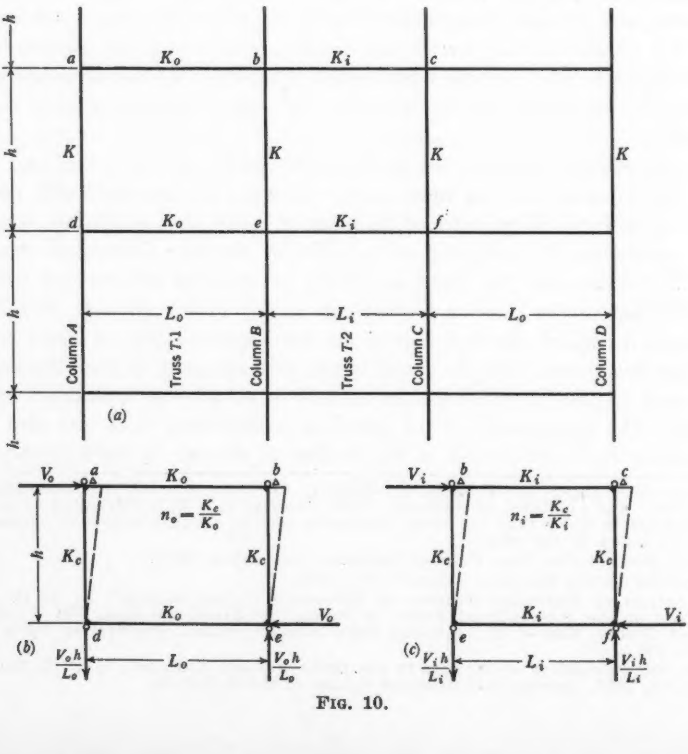


FIG. 10.

Since the value of Δ must be the same for each panel in the story, $V_i\ (1 + n_i) = V_o\ (1 + n_o)$; or,

$$V_i = V_o \left(\frac{1 + n_o}{1 + n_i}\right) \qquad (3)$$

and if V is the total shear in the story then, $2\ V_o + V_i = V$, from which,

$$V_o = \frac{1 + n_i}{3 + n_o + 2\ n_i}\ V \qquad (4a)$$

and,

$$V_i = \frac{1 + n_o}{3 + n_o + 2\ n_i}\ V \qquad (4b)$$

By referring to Fig. 10(b) and 10(c) it can be seen that the increment of stress in Column A will be,

$$T_A = \frac{V_o\ h}{L_o} = \frac{V\ h}{L_o}\left(\frac{1 + n_i}{3 + n_o + 2\ n_i}\right) \qquad (5)$$

and in Column B,

$$T_B = \frac{V_i\ h}{L_i} - \frac{V_o\ h}{L_o} = \frac{V\ h}{3 + n_o + 2\ n_i}\left(\frac{1 + n_o}{L_i} - \frac{1 + n_i}{L_o}\right) \qquad (6)$$

In the cantilever method, $T_B = \dfrac{L_i}{2\ L_o + L_i}\ T_A$; or,

$$\left(\frac{1 + n_o}{L_i} - \frac{1 + n_i}{L_o}\right) = \frac{L_i}{2\ L_o + L_i}\left(\frac{1 + n_i}{L_o}\right) \qquad (7)$$

If $i = \dfrac{I_i}{I_o}$ and $c = \dfrac{L_i}{L_o}$, then from Equation (7):

$$i = \frac{2\ c^2\ (c + 1)}{2 + c - \dfrac{1}{n_o}\ (2\ c^2 + c - 2)} \qquad (8)$$

In the portal theory, $T_B = 0$; or, $\left(\dfrac{1 + n_o}{L_i} - \dfrac{1 + n_i}{L_o}\right) = 0$, from which,

$$i = \frac{c^2}{1 - \dfrac{1}{n_o}\ (c - 1)} \qquad (9)$$

The results of Equations (8) and (9) are shown graphically in Fig. 11 in which curves are shown for various values of n_o. These curves show that the rigidity of the columns as compared with the girders has considerable importance in determining the value of i, particularly for values of c greater than 0.8. Furthermore a probable curve for the values of i, that would be obtained if the beams were designed for the same unit, uniform, vertical load, has been shown in Fig. 11. This curve is more of a third power of c than a second power.

From a study of the relative positions of the curves in Fig. 11 it would seem that the following general statements can be made, most of which corroborate the conclusions of Messrs. Witmer and Bonner:

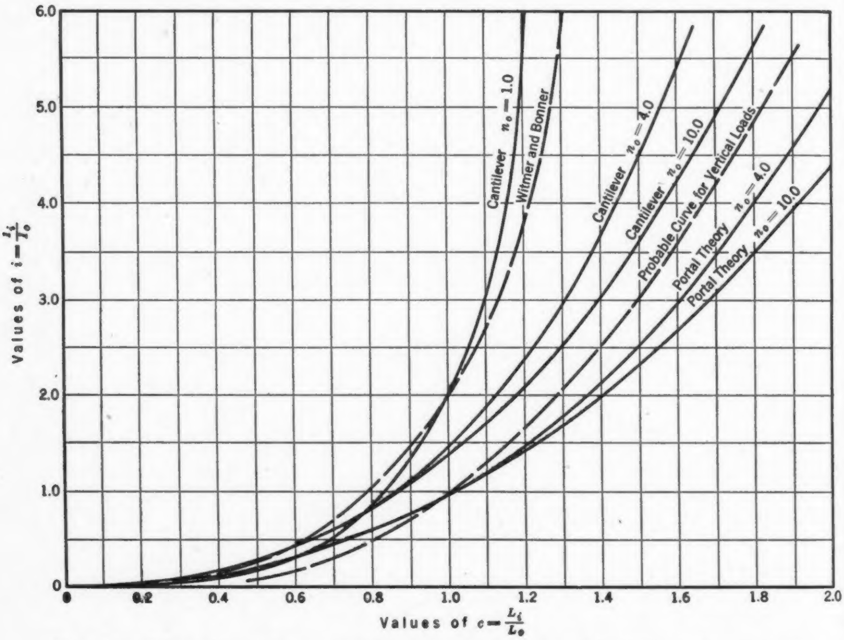


FIG. 11.—RELATION OF SPAN AND MOMENT OF INERTIA OF GIRDERS FOR CANTILEVER AND PARTIAL METHODS OF ANALYSIS.

(1) For values of c less than 0.6, the portal theory will probably be most correct for girders that are designed for vertical loads only, but that small changes in the rigidity of the girders or columns will tend to produce considerable percentage changes in the results.

(2) For values of c between 0.6 and 1.2, the portal theory will be most correct for all rigidity ratios of girders and columns unless the center girders are arbitrarily increased in size.

(3) For values of c greater than 1.2, the cantilever theory will be most correct if the columns are much more rigid than the outer girders; otherwise, neither theory will give great accuracy.

(4) For values of n_o less than 1.5, the rate of change of i with respect to c is too rapid to make a numerical comparison of much use. The reason for this difference in accuracy can be most easily seen by referring to Fig. 12 in which the variation of i with respect to n_o is given for a particular value of c. From this curve it can be seen that any conclusions that might be made from the portion of the curve for values of n_o less than 1.5 would be very uncertain in view of the practical difficulty of calculating the true value of n_o.

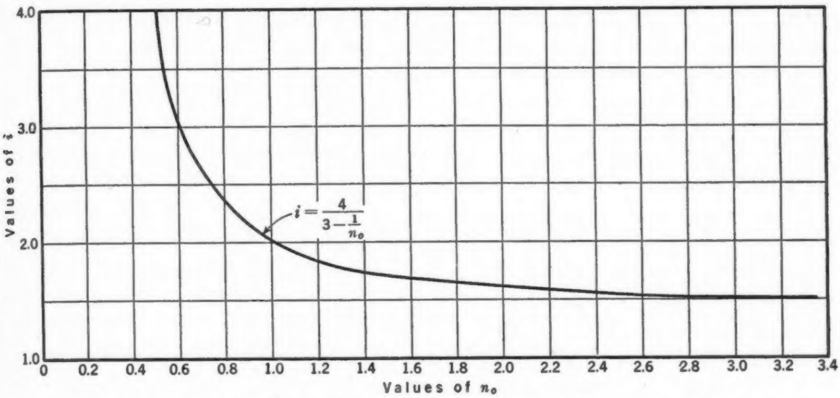


FIG. 12.—RELATION OF i AND n_0 FOR CANTILEVER METHOD; $c = 1$.

In conclusion, the writer wishes to make the usual observation that the effect of the direct stress in the columns may modify the final results both for the hypothetical frame and for the model. In this respect a model that is proportioned according to the dimensions of the actual structure will have considerable advantage, but there may be greater disadvantages. There is no general law that any type of structure will inherently follow, except the law of minimum energy, but perhaps certain regular types of structures can be forced into some definite group without undue loss of economy. Special or irregular structures, of course, can never be simplified in such a manner.

L. J. MENSCH,[7] M. AM. SOC. C. E. (by letter).[7a]—When the endeavor of mathematical analysts failed to provide engineers with an easily understood and practical working rule for the design of wind-bracing bents, theory became of limited value and investigators returned to the difficult and expensive path of model research. Models, often of full size, have been used since the dawn of civilization. What amount of experimenting was necessary before the Phœnicians had learned to cut from round trees of Lebanon the strongest rectangular beams for the Temple of Solomon? What proportion of the diameter was the width to be, and what proportion the depth? It is certainly very curious that they knew the correct answer probably 3 000 yr ago; in the present day any smart structural engineer, just out of college, can solve this problem in a few minutes by mathematical analysis.

Is it really easier to find a solution by model research alone? Does it take less perception or less theoretical knowledge to do so successfully? It is certainly more expensive and beyond the reach of most engineers and the likelihood of making errors is nearly, if not quite, as great as when analysis alone is used. Unless such tests are thoroughly analyzed, without any endeavor to fit the analysis by illogical means to the tests, experimenters will find that their patient and valuable work is received with indifference by the profes-

[7] Civ. Engr. and Constructor, Chicago, Ill.

[7a] Received by the Secretary February 13, 1936.

sion. The reverse holds also true, however; when a mathematical analyst investigates a new problem his endeavors are only of little value to the profession unless he substantiates his findings by careful tests.

The writer proposes to show that the models described in this paper were not as perfect as they should have been, a fact evidently known to the authors, as they wisely stated that their results should be accepted more to indicate trends of the reactions for the relative proportions of columns and girders.

The portal and cantilever methods of finding wind stresses are crude guesses at solutions and may be compared with the method used by a better carpenter in designing joists for frame buildings. He computes the load on the joist and selects, from a table of strength of wooden beams, the joist he needs. He ignores the fact that the joist may be continuous over three or four supports. These methods may also be compared with the habit of structural engineers in designing steel trusses for mill buildings. Instead of designing them as rigid frames, the trusses are universally considered as simply supported at the columns and, for good luck, are braced to the columns.

Meanwhile, inconveniences were experienced in tall buildings due to swaying under wind. The first instance that came to the writer's notice were complaints of draftsmen, working in the oldest 17-story building in Chicago, Ill. Engineers in that building moved to lower floors, preferably below the tenth. Investigations were made, and the amplitude of the vibrations were found to be of the order of $\frac{1}{16}$ in.; the building was strengthened by knee-braces at some of the columns and vibrations were greatly reduced. In another 16-story building draftsmen could not work on the tenth floor of the building when a high wind was blowing and the partitions in that building were vibrating so visibly that clerks moved their desks many feet away. When high hotel and apartment buildings came into general use, complaints were heard from occupants that they could not sleep on account of window weights hitting the window boxes and because water was spilling out of glasses and wash bowls, and pictures and chandeliers were swinging in a high wind.

It became apparent that a more accurate analysis was needed for the design of tall building frames, and it is to the credit of Albert Smith, M. Am. Soc. C. E., that he first published[8] a practical rule for the design of such frames, based on a laborious analysis by Castigliano's theorem of least work; his rule was a great improvement over the portal and cantilever methods, but was received indifferently by those engineers who did most of the designing of tall building frames, possibly on account of the labor involved in checking the basic calculations.

W. M. Wilson and G. A. Maney, Members, Am. Soc. C. E.[9], analyzed a 20-story building frame by assuming the rotations of the joints as unknowns in the elastic equations and gave their analysis the novel and rather unfortunate name of "slope-deflection theory." Their method and that of Mr. Smith were just as correct as Clapeyron's method of analyzing continuous beams,

[8] *Journal*, Western Soc. of Engrs., April, 1915.

[9] "Wind Stresses in the Steel Frames of Office Buildings", *Bulletin No. 80*, Eng. Experiment Station, Univ. of Illinois, 1915.

but were not used as much as the latter, because, in the nature of the case, these methods are too laborious for practical use. Furthermore, Messrs. Wilson and Maney deduced from their research a rule for simplifying the analysis, which is more correct for varying sections and spans than Mr. Smith's simple rule, but still too complicated for most practising engineers. The result of these two most important endeavors was that most engineers used the method of least work, namely, the crude and easily understood portal and cantilever methods.

Although it was long known that longitudinal deformation of the columns under wind, and also under dead load and live load, might produce considerable changes of stress in the girders of tall frames, and was found by Messrs. Wilson and Maney to amount to only a small percentage in a 20-story building, Henry V. Spurr, M. Am. Soc. C. E., in 1930, introduced an improved cantilever method[9] in which the longitudinal deformation of the columns was made the corner stone of his method of computing wind moments, neglecting at the same time other important factors, such as the law of least cost of a structure.

In 1931, the writer undertook to write a paper[10] on deflection and vibrations of tall building frames under wind and, at the same time, to give a simple working rule for the computation of wind stresses which the busy engineer could use. He first faced this problem in 1910 when he designed column supports for high elevated tanks of reinforced concrete without diagonal braces. The frames were, in fact, two column bents of several stories. When computing the deflection and rotations of the joints he noticed that the members far away from a joint had little influence on stresses produced there, and that

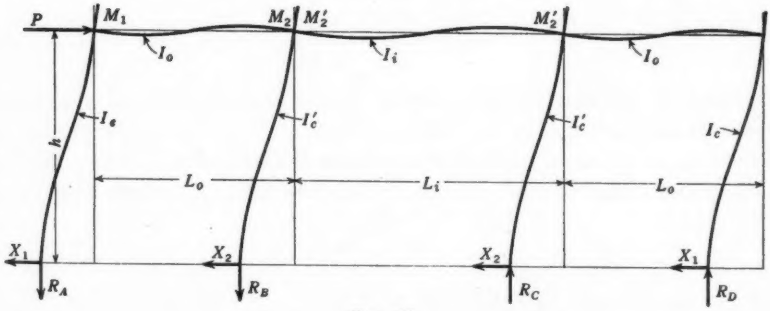


FIG. 13.

fairly accurate results could be obtained for bents with any number of spans and stories by the following assumptions:

(1) The point of contraflexure of all columns, except in the lowest story, is in the center of the story height;

(2) Where the values of stiffness of the girders, $\frac{I}{L}$, adjoining an inside

[9] "Wind Bracing", by Henry V. Spurr, N. Y., McGraw-Hill Book Co., 1930.

[10] "Deflections and Vibrations in High Buildings", by L. J. Mensch, *Proceedings*, Am. Concrete Inst., Vol. 28, 1932, p. 387.

column are alike, the wind moments in the girders at each side of the column at the joint are alike;

(3) Where the values of stiffness of the girders adjoining inside columns are not alike, the moments in the girders induced by the deflection of the columns are proportionate to the stiffness of the girders.

For the case shown in Fig. 13, which embraces all cases with four columns treated in the present paper, the writer deduced the formulas in the paper[10] mentioned:

$$X_1 = \frac{P}{4}\left(n_i + \frac{6}{1+n}\right) \div \left(\frac{n_o + n_i}{2} + \frac{6 + 3\,N}{1 + N}\right) \quad \text{.......(10)}$$

$$X_2 = \frac{P}{2} - X_1 \quad \text{........................(11)}$$

$$M_1 = X_1\,h \quad \text{........................(12)}$$

$$M_2 = \frac{X_2\,h}{1 + N} \quad \text{........................(13)}$$

$$M'_2 = \frac{X_2\,h\,N}{1 + N} \quad \text{........................(14)}$$

$$R_A = \frac{M_1 + M_2}{L_o} \quad \text{........................(15)}$$

and,

$$R_A + R_B = \frac{2\,M'_2}{L_i} \quad \text{......................(16)}$$

in which, in addition to the notation of the main paper and the symbols illustrated in Fig. 13, P = the entire wind load above the middle of any story considered; n = ratio of stiffness between girders and columns (the subscript, i, denoting "interior bay" and the subscript, o, "outside bay"); and,

$$N = \frac{L_o\,I_i}{L_i\,I_o} \quad \text{......................(17)}$$

When all columns are alike and when the values of stiffness of the girders are all alike, Equation (10) reduces to,

$$X_1 = \frac{P}{4}\,\frac{n_o + 3}{n_o + 4.5} \quad \text{......................(18)}$$

and,

$$M_2 = M'_2 = \frac{X_2\,h}{2} \quad \text{......................(19)}$$

In the lower part of very tall buildings the stiffness of the columns is often ten times as great as that of the girders, and no great error is made by

assuming n_o and n_i as zero in Equations (10) and (18), and Equation (10) becomes,

$$X_1 = \frac{P}{4}\,\frac{2}{2 + N} \quad \text{.........................(20)}$$

which means that the shear and reactions in the columns are mainly dependent on the relative stiffness of the girders.

When it is desired to find the relative stiffness of the girders, which would satisfy Mr. Spurr's idea of proportionate column stress reactions, one can proceed in this case as follows: Substitute Equation (20) in Equation (11),

$$X_2 = \frac{P\,(2 + 2\,N)}{4\,(2 + N)} \quad \text{......................(21)}$$

substituting Equation (20) in Equation (12), and Equation (21) in Equation (13),

$$M_1 = M_2 = \frac{2\,P\,h}{(2 + N)\,4} \quad \text{...................(22)}$$

which means that the point of inflection of the girders, where very heavy columns are used, is in the center of the girders. Consequently, Equations (14), (15), and (16) become:

$$M'_2 = \frac{X_2\,h\,N}{1 + N} = \frac{2\,N}{(2 + N)}\,\frac{P\ h}{4} \quad \text{...............(23)}$$

$$R_A = \frac{M_1 + M_2}{L_o} = \frac{4}{(2 + N)}\,\frac{P\ h}{4\ L_o} \quad \text{...............(24)}$$

and,

$$R_A + R_B = \frac{2\,M'}{L'_i} = \frac{4\ N}{(2 + N)}\,\frac{P\ h}{4\ L_i} \quad \text{...............(25)}$$

When $L_o = L_i$, $R_A + R_B = N\ R_A$; and,

$$R_B = (N - 1)\ R_A \quad \text{......................(26)}$$

Mr. Spurr's assumptions of a straight-line deformation of a transverse section of a frame may be expressed mathematically in this case of equal spans by $\frac{R_A}{A_o} \div \frac{R_B}{A_i} = 3$; but, from Equation (26), $\frac{R_A}{R_B} = \frac{1}{N - 1}$; and, therefore,

$$N = 1 + \frac{A_i}{3\ A_o} \quad \text{.......................(27)}$$

in which A_o = the cross-section area of the outside column; and A_i = the cross-section area of the inside column. In other words, regardless of the requirements of dead and live load and wind-bending stresses, the section of the central girder must be increased which in most cases invites a waste of

materials, especially in the lower part of the frames. The authors' models did not show this relation, as their columns were not much stiffer than the girders.

For young engineers who have done much research in the analysis of wind-bracing bents and who may be still doubtful as to the validity of Equation (10) on account of the writer's Assumptions (2) and (3), the following formula is presented, but is not needed in actual practice:

$$X_1 = \frac{P}{4}\left\{\frac{2 + \frac{n_t}{3} + \frac{2\,n_t\,N}{3}}{2 + \frac{n_o}{6}\left(1 + \frac{I_c}{I'_c}\right) + N\left[1 + \frac{n_o}{3}\left(1 + \frac{I_c}{I'_c}\right)\right]}\right\} \quad \ldots (28)$$

The results of Equation (10) vary only a small percentage from those of Equation (28), whereas Equation (18) will give results that may vary 5% in the lower part of a tall building and as much as 20% at the very top.

Clear spans are the governing factors[10] in the design of building frames and the use of theoretical spans causes the computed stresses to be often 10% to 30% greater than they appear from tests to destruction. When mathematical analysts, using an "exact" theory, start with an assumption which misleads them from 10 to 30% is it to be wondered that practical engineers do not pay much attenion to their work? Equations (10), (18), (20), (21), and (28) do not, as a rule, give results that differ greatly whether clear or center-to-center spans are used. The moments in the columns and girders, however, vary greatly because of the shorter lengths of the clear spans, and the deflections and vibrations are also much smaller.

The writer wishes to repeat that he has every reason to believe that his formulas give results as reliable as the continuous beam formulas, provided, of course, that the joints are properly designed. In fact, he measured the deflection and rotation of joints on two model frames of 6-story and 13-story buildings, made of brass rods, soldered at the joints, and found a very good agreement with his analysis; and when his formulas give results that differ with the tests of the authors, he does not hesitate to state that their models did not have proper joints.

It will be interesting to check Cases *A*-1, *A*-2, *A*-1*a*, and *A*-1*c* of the paper by Equations (10) to (28). For Cases *A*-1 and *A*-1*a*, all columns are alike, all girders are alike, $h = 2\frac{1}{2}$ in., and $L_o = 4$ in. for center-to-center spans and, according to the authors, the clear spans are to be taken as $\frac{1}{2}$ in. less. The ratio, n, is $\frac{2.5}{4} = 0.625$ when theoretical spans are used, and n is $\frac{2}{3.5} = 0.571$ when clear spans are used. In both cases, $N = 1$. From Equation (28), $X_1 = \frac{P}{4}\,\frac{2 + n}{2 + \frac{n}{3} + 1 + \frac{2\,n}{3}} = 0.181\ P$ and $0.18\ P$, respectively. Equations (10) and (18) give identical values ($0.177\ P$ and $0.176\ P$, respectively).

Adopting the value of $0.18\,P$ for X_1, $X_2 = \dfrac{P}{2} - 0.18\,P = 0.32\,P$. From Equation (12), $M_1 = 0.18\,P\,h$ for center-to-center spans and $0.18\,\dfrac{3.5 P\,h}{4}$ for clear spans. Furthermore, from Equations (13) and (14), $M_2 = M'_2 = \dfrac{0.32\,P\,h}{2}$ and $\dfrac{0.32\,P\,h}{2}\dfrac{3.5}{4}$, respectively. Equation (15) yields: $R_A = \dfrac{P\,h \times (0.18 + 0.16)}{3.5} \times \dfrac{3.5}{4} = 0.85\,P\,h = 0.2125\,P$, and Equation (16), $R_A + R_B = \dfrac{0.32\,P\,h}{3.5} \times \dfrac{3.5}{4} = 0.08\,P\,h = 0.08\,P \times 2.5 = 0.200\,P$; or, $R_B = 0.200\,P - 0.2125\,P = -\,0.0125\,P = -\,0.0588\,R_A$, against $-\,0.15\,P$ and $-\,0.19\,P$ from Models A-1, and A-1a.

Case A-2 was like Case A-1 except that all center girders were twice as stiff as the outside girders; Hence, $N = 2$, whereas $n_o = 0{:}571$, as before. From Equations (10) to (16), respectively: $X_1 = \dfrac{P}{4} \times \dfrac{0.571 + 2}{0.571 + 4} = 0.1406\,P$; $X_2 = 0.3594\,P$; $M_1 = X_1\,h \times \dfrac{3.5}{4} = 0.1406\,P\,h \times \dfrac{3.5}{4}$; $M_2 = \dfrac{X_2\,h}{1 - N} \times \dfrac{3.5}{4} = 0.1198\,P\,h \times \dfrac{3.5}{4}$; $M'_2 = \dfrac{X_2\,h\,N}{1 + N} \times \dfrac{3.5}{4} = 0.2396\,P\,h\,\dfrac{3.5}{4}$; $R_A = \dfrac{0.1406 + 0.1198}{3.5} \times P\,h \times \dfrac{3.5}{4} = 0.163\,P$; $R_A + R_B = \dfrac{2 \times 0.2396}{3.5} \times P\,h \times \dfrac{3.5}{4} = 0.299\,P$; and $R_B = 0.299\,P - 0.163\,P = 0.136\,P = 0.835\,R_A$. The model for this case showed that $R_B = 0.33\,R_A$, which is a proof in the writer's mind that the connections in this case were not very effective.

In Case A-1c all girders and outside columns were as in Case A-1 except that the inside columns were twice as thick and their stiffness eight times as great; hence, $n_o = 0.571$; $n_i = \dfrac{0.571}{8} = 0.0714$; and $N = 1$.

From Equations (10) to (16), respectively: $X_1 = \dfrac{P}{4} \times \dfrac{0.571 + 3}{0.321 + 4.5} = 0.1855\,P$; $X_2 = 0.3144\,P$; $M_1 = 0.1855\,P\,h \times \dfrac{3.5}{4}$; $M_2 = M'_2 = 0.1572\,P\,h \times \dfrac{3.5}{4}$; $R_A = \dfrac{0.1855 + 0.1572}{3.5} \times P\,h \times \dfrac{3.5}{4} = 0.0857\,P\,h$; $R_A + R_B = \dfrac{2 \times 0.1572}{3.5} \times P\,h \times \dfrac{3.5}{4} = 0.0786\,P\,h$; and $R_B = 0.0786\,P\,h - 0.0857\,P\,h = -\,0.0071\,P\,h = -\,0.0828\,R_A$, which is very close to the result found by the writer for Cases A-1 and A-1a, and, this time, more in agreement with the model test reported in the paper.

From the foregoing examples one may conclude that the authors were correct in their statement that the tests on the models showed the trend of reactions, but their values differed considerably with that obtained by analysis.

No answer was given for the reactions for a horizontal load applied at the first-floor level which often varies considerably from the upper floors.

It would have been of great interest if, on one welded model, tests had been made on the period and amplitudes of vibrations and if the amplitudes had been compared with those of a solid bar of the same material having the same period of vibration.

GILBERT MORRISON,[11] M. AM. SOC. C. E. (by letter).[11a]—Information of much interest to designing engineers, on the subject of lateral forces, on tall building bents, has been presented in this paper. The results of the behavior of the many different combinations of bent structures, under tests shown by the authors, should be of material assistance in predetermining how similar structures are likely to act under lateral forces.

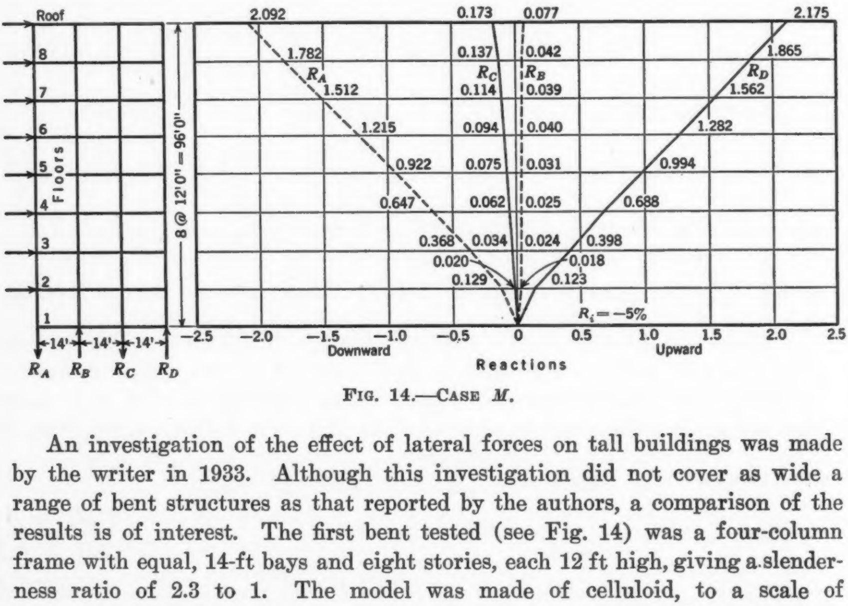


FIG. 14.—CASE M.

An investigation of the effect of lateral forces on tall buildings was made by the writer in 1933. Although this investigation did not cover as wide a range of bent structures as that reported by the authors, a comparison of the results is of interest. The first bent tested (see Fig. 14) was a four-column frame with equal, 14-ft bays and eight stories, each 12 ft high, giving a slenderness ratio of 2.3 to 1. The model was made of celluloid, to a scale of 1 in. = 4.0 ft. The apparatus used in the test was a Beggs deformeter. In this bent all girders were of the same size, having a moment of inertia of 1 728. The inner and outer columns of any floor were the same; but were increased in size downward for each two stories. The value of I for columns in the first and second stories was 2 920; for the third and fourth-story columns, $I = 2.835$; for the fifth and sixth-story columns, $I = 1\,728$; and for the seventh and eighth-story columns, $I = 1\,028$. All members of the model were proportioned to the cube root of their moment of inertia and the aforementioned scale. A lateral force of unity was acting at each floor above the first.

[11] In Chg. of Structural Design, Cooper Union Inst. of Technology, New York, N. Y.
[11a] Received by the Secretary February 14, 1936.

In Fig. 14, negative reactions, acting downward, are plotted to the left and positive reactions, acting upward, are plotted to the right. The summation of the abscissas from the top down to any floor, for any column, indicates the value of the vertical reaction for that column, for the unit loads applied at those floors. Hence, the summation of all the abscissas, from top to bottom, gives the value of the vertical reaction for that column, with lateral forces acting at all floors simultaneously, proper care being given to positive and negative signs. This shows that the vertical reaction for Column $A = -8.667$, for Column $B = +0.296$, for Column $C = -0.709$, and for Column $D = +9.087$, indicating an almost exact balance.

In this case, as in that of the paper, it is seen that the interior reaction is opposite in character from the exterior reaction on the same side of the bent, and also of relatively small magnitude, the average interior reaction being approximately -5% of the exterior reaction. This is less than the -15% shown for the authors' Case *A*-1. It should be noted that the only difference between the bents treated by the authors and the writer, is that in the former all the columns were kept the same size throughout the full height of the structure, whereas, in the latter, the column sizes were changed every two floors.

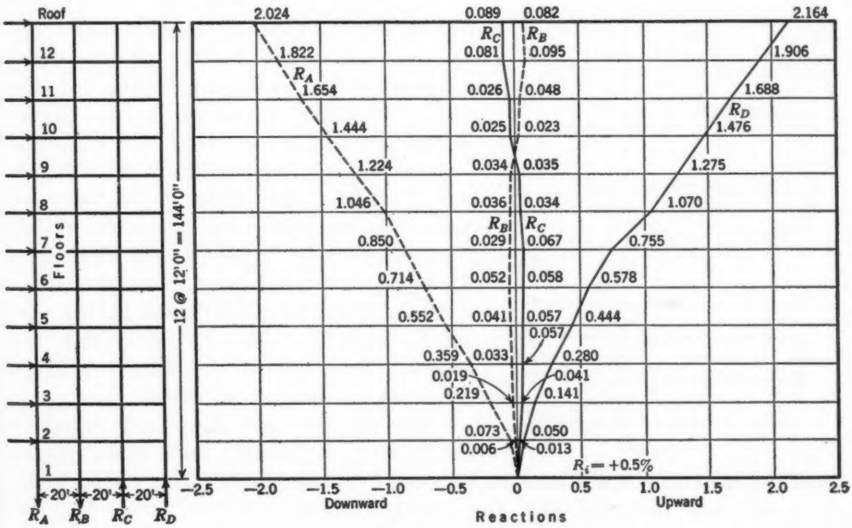


FIG. 15.—CASE *N*.

The results of the second bent tested by the writer (see Fig. 15) was also a four-column frame with equal, 20-ft bays and 12 stories each 12 ft high, giving a slenderness ratio of 2.4 to 1. This model was also made of celluloid, to a scale of 1 in. = 4 ft and tested with the Begg's deformeter. In this case all floor girders were of the same section, being 18-in. I-beams at 55 lb, with $I = 889.9$. However, the roof girders were 16-in. I-beams at 45 lb.,

with $I = 583.3$. Like Case *M* column sections were changed at each two succeeding stories. Both outer columns were alike and both inner columns were alike in any story. A complete section of the bent and a plan, showing sizes of different members and their *I*-values, is shown in Fig. 16. Inner column *I*-values are greater than outer column *I*-values; but not by any fixed ratio. For instance, for Stories 11 and 12, the *I*-value for inner columns is 33% greater than for outer columns, whereas, for Stories 9 and 10, the *I*-value for inner columns is only 12% greater than that for outer columns—and so on, down to the bottom, there is no fixed ratio. It is seen that sections were not picked for any fixed relation to *I*-values; but that both columns and girders were designed only for vertical loads, thus fixing the *I*-values.

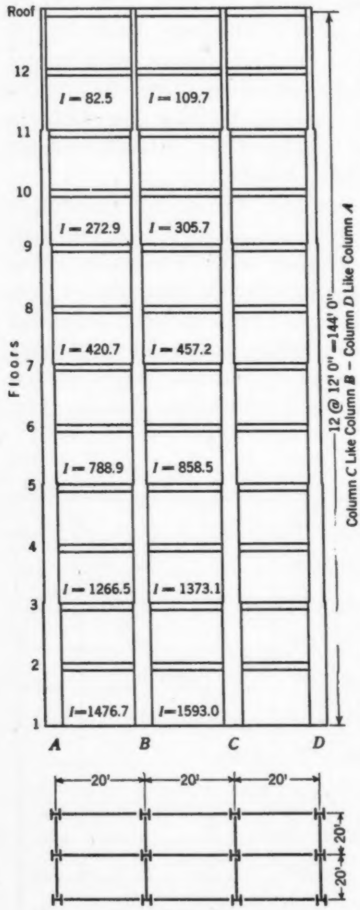


FIG. 16.

The authors' case most similar to this, is Case *A*-1*c*, with the exception that the latter carries the same size of columns for the entire height of the bent and keeps the relation of the *I*-values of the inner columns eight times the *I*-value of the outer columns; the writer's case shows actual sizes in an existing typical building bent. However, a study of Fig. 15, will show that the character of the reactions was the same on each side of the bent, contrary to that shown for Case *A*-1*c*. This test shows an almost exact agreement with the distribution indicated by the portal theory. It should be noted that the reaction for both inner columns changes direction in the ninth story.

The upward and downward reactions in Column *B* balance each other within 0.002, whereas the same items in Column *C* balance each other within 0.141. It should also be noted that the average interior reaction is + 0.5% of the exterior reaction. The total vertical reactions for Column $A = -11.981$, for Column *B*, − 0.002, for Column *C*, + 0.141, and for Column *D*, + 11.827, again showing an almost exact balance.

In view of these tests and the tests described in the paper, the writer is in agreement with the authors' statement that, if a structure is designed for vertical loads only, with *I*-values of girders assumed in proportion to the square of the span length, the reactions are in very close agreement with distribution determined by the portal theory. However, it is believed that a bent

designed for vertical loads only, is not a typical frame from which to verify the application of any theory as to the distribution of lateral forces because a bent designed for both vertical and lateral loading, would undergo considerable change in structural requirements over one designed for vertical loads only. The writer does not feel that he would care to support, without reservations, the authors' statement that a normal variation in column sizes, increasing toward the bottom, would not greatly vary the general relations shown.

It should be noted that Cases *A*-1, *A*-2, *A*-3, *A*-9, and *A*-17, of the paper (in which *I*-values of center bays are increased by 2, 3, 9, and 17, respectively) indicate an increase in R_t, which was to be anticipated, because of the additional stiffness given; but the additional increases in R_t are not proportional to the increases in moments of inertia, which is clearly shown in Fig. 3.

In Case *E*-1.41 it would appear that the reactions, *A*, *B*, and *C*, act upward, whereas Reaction *D* acts downward. If this is correct, there appears to be about 20% variation between upward and downward reactions.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MODERN CONCEPTIONS OF THE MECHANICS OF FLUID TURBULENCE

Discussion

BY CHESLEY J. POSEY, JUN. AM. SOC. C. E.

CHESLEY J. POSEY,[24] JUN. AM. SOC. C. E. (by letter).[24a]—In publishing this excellent paper the author has performed a service to the Engineering Profession. It summarizes, in a lucid manner, the progress made upon the important problem of fluid turbulence. The writer has recently developed an analytical approach to certain of the unsolved phases of the problem to which the author calls particular attention. Although the writer's studies are as yet in a tentative stage, it seems worth while to outline them briefly in discussion.

The author points out that one of the chief difficulties in the study of fluid resistance in rough pipes is the lack of a satisfactory measure of the roughness. The measure, ϵ, representing the radial dimension of roughness particles, is apparently only satisfactory when the particles are geometrically similar. The curves shown in Fig. 10, for artificially roughened pipe, cannot be expected to apply to other pipe as the friction factor, at large values of R, is a function not only of $\frac{r_0}{\epsilon}$, but also of other unknown factors dependent upon the geometry of the roughness. In mathematical terms:

$$f = \Phi\left(\mathsf{R}, \frac{r_0}{\epsilon}, \,?\,, \,?\,, \,..\right) \qquad (69)$$

It is the writer's intention to point out what may be the nature of the unknown factors. The form of the function must remain to be determined experimentally.

Fig. 21 is a diagrammatic longitudinal section through the wall of the pipe, showing the inside surface. The section is long enough to include a representative sample of the "roughness." The dimension, ϵ, is of the extreme

NOTE.—The paper by Hunter Rouse, Assoc. M. Am. Soc. C. E., was published in January, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[24] Asst. Prof. of Mechanics and Hydraulics, State Univ. of Iowa; and Assoc. Engr., Iowa Inst. of Hydr. Research, Iowa City, Iowa.

[24a] Received by the Secretary February 29, 1936.

radial variation in the pipe's inner surface. Two curves are plotted above the "roughness" curve, representing its first and second derivatives with respect to the distance along the pipe. The maximum ranges of the two curves may

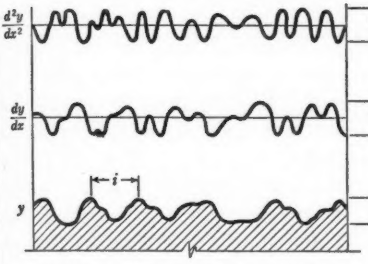


FIG. 21.—LONGITUDINAL SECTION THROUGH PIPE WALL, SHOWING ROUGHNESS, AND DERIVATIVE CURVES.

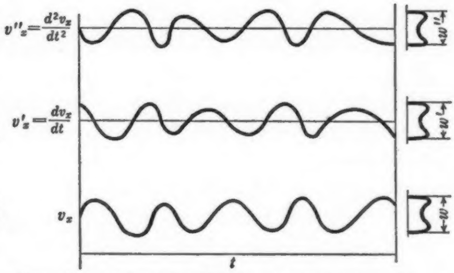


FIG. 22.—INSTANTANEOUS VELOCITY AS A FUNCTION OF TIME, AND DERIVATIVE CURVES.

be termed ϵ' and ϵ'', as shown. The first is obviously dimensionless, whereas the second must be multiplied by a length parameter to be dimensionless. The appropriate parameter would seem to be the average distance between successive roughness particles. Higher derivatives might be taken, but when it is noted that ϵ' is a measure of the maximum slopes of the roughness, and that ϵ'' is a measure of the sharpness of its points, it seems unlikely that higher derivatives will be needed to characterize the roughness satisfactorily. Thus:

$$f = \Phi\left(\mathsf{R}, \frac{r_0}{\epsilon}, \epsilon', i_{av}\,\epsilon''\right) \qquad (70)$$

It is questionable whether experiments to determine the form of Φ should include unnatural geometrical roughnesses, but whether artifical or natural roughnesses are investigated the experiments should include a measurement of the roughness as outlined herein. The technique of making such a measurement would require taking a precise profile along a longitudinal section of the pipe by use of the recently developed "profilograph", or by other means. The derivative curves could then be constructed graphically. In commercial pipes the roughness measured in a transverse direction might be different from that measured in a longitudinal direction. It is to be hoped that the transverse roughness, where different, will have a comparatively minor effect upon the frictional resistance to flow in the pipe.

In the more general problem, there is a need for a definite measure of turbulence. It is evident that the complete picture of velocity distribution in a turbulent region at any instant can never be exactly repeated; yet the turbulent stream may be "steady" in the sense that certain measures do remain constant, statistically. For example, the average velocity at a point during successive intervals of time may remain constant if the intervals of time are long enough. The average velocity, however, is unsatisfactory as a measure of the turbulence. To arrive at a definition of the statistical measures which may be expected to provide a sufficiently precise measure of the turbu-

lence, consider the variation of the x-component of the velocity at a point in a turbulent stream. The x-direction is understood to be the direction of the mean velocity. Fig. 22 shows an imaginary curve for v_x as a function of t. Above it are the first two derivative curves, v'_x and v''_x. At the right end of each curve is represented the frequency distribution of ordinates during equal infinitesimal increments of time. The x-component of the turbulence at one point may be said to be identical with that at another if these distributions of v_x, v'_x, and v''_x are identical at each. Distributions from higher order curves are omitted arbitrarily; perhaps that of the second order is unnecessary. The same procedure can be applied to v_y and v_z. The use of six or nine frequency distributions to measure the turbulence at a single point seems hopelessly complicated, but in many cases correlations will reduce the number, and it may be that the double-moded distributions of Fig. 22 can be represented with sufficient accuracy for experimental work by the ranges, w, w', and w''. These ranges can be reduced to dimensionless form by dividing by appropriate parameters, but this should not be done except in comparisons of geometrically similar cases.

The measure of turbulence suggested by the writer bears somewhat the same relation to von Kármán's universal relation for the mixing length as does the Eulerian viewpoint to the Lagrangian viewpoint in classical hydrodynamics. The concept of mixing length is perhaps better adapted to studies of energy loss than that of the writer, but the writer's concept would seem susceptible to more direct experimental application. Study of each may yield information valuable in the development of the other.

The following inadvertant errors, reported by the author, are to be corrected when the paper is published in *Transactions*: In Equation (20), change the quantity in parenthesis to $\left(1 - \frac{y}{r_0}\right)$; in the caption for Fig. 11, reference to Fig. 8 should be changed to Fig. 10; in the line following Equation (43), change "directly" to "inversely"; delete the entire caption for Fig. 16; in the caption for Fig. 17, change "rough" to "smooth"; the correct caption for Fig. 18 is "Universal Velocity Distribution for Rough Pipes."

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY MESSRS. ROBERT FOLLANSBEE, AND LEROY K. SHERMAN

ROBERT FOLLANSBEE,[2] M. AM. SOC. C. E. (by letter).[2a]—The Committee's recommendations interest the writer, who for many years has been gathering data on floods in connection with the regular work of the Water Resources Branch of the U. S. Geological Survey.

The 1935 flood on the Republican River emphasized pointedly the need for a continuation of the tabulation of flood data. That flood, caused by unprecedented rainfall in the Western Plains region, had a peak flow on the upper river about ten times greater than any historic flood on that river covering a period of 70 yr. The longest stream-gaging record on that river, in Nebraska, was that at Superior-Bostwick from 1896 to 1914, during which time the maximum discharge was 24 500 cu ft per sec, and occurred in 1914. A relation between the peak flow of the 1935 flood, which amounted to about 240 000 cu ft per sec in that section, and previous floods within the past 70 yr, was shown near Oxford, Nebr. In 1865, a family settled near the river, and has occupied the same site ever since. At no time prior to 1935 had flood waters ever approached the house. Attached to it is a lean-to, more than 10 ft high, which was completely submerged during the peak of the 1935 flood. This indicates that the latter flood was at least 12 ft higher than any previous flood in 70 yr and, considering the fact that the 12-ft excess height represented very largely overflow area, it is evident that previous floods were so much smaller as to be comparable with the 1914 peak of 24 500 cu ft per sec.

The investigation of this flood brought out a point that has been noticed in studies of previous floods; that is, that in the Western Plains region, at least, U. S. Weather Bureau precipitation stations are too few and far between to give indication of the quantity of rainfall causing the flood. The

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[2] Dist. Engr., U. S. Geological Survey, Denver, Colo.

[2a] Received by the Secretary March 21, 1936.

chief cause of the Republican River flood was rainfall of cloudburst proportions covering a relatively small area in Eastern Colorado and Western Nebraska, an area containing no rainfall stations. As is frequently the case during extraordinary rainfall, ranchers measured that rainfall in various receptacles standing in their yards. A personal search for such data brought to light many records of rainfall ranging from 4 to 21 in. during the night preceding the flood. When plotted on a map these records were found to be so consistent as to warrant a belief in their substantial accuracy.

The subject of cloudburst floods is a live one in Colorado and Wyoming, and floods of that character have been investigated by the writer, and the areas chiefly affected, have been defined with a fair degree of accuracy[3]. Much more work, however, remains to be done in order that the maximum unit run-off to be expected from floods of that character can be determined with sufficient accuracy to be of real value to the Engineering Profession. It is hoped that means may be found to carry out the recommendations of the Committee.

LeRoy K. Sherman,[4] M. Am. Soc. C. E. (by letter).[4a]—The Committee has performed a valuable advisory service for the Federal Government in its preparation of *Water Supply Paper 771*, "Floods in the United States—Magnitude and Frequency." This paper is an outstanding contribution toward flood-relief measures.

In five recommendations relative to further flood-protection investigations, the Committee points out several important studies. It stresses the further collection of basic data. This collection and the recording of rainfall and flood run-off must be a continuing operation. However, in the writer's opinion, the collection of data is no longer the most important phase. A voluminous record is already available relating to floods and flood-producing storms for a period of seventy years. More recently, this inventory includes the "308" reports of the U. S. Corps of Engineers, State Planning Board reports, and the reports of the Water Resources Section of the National Resources Committee.

The future now calls for analysis and study of these data and this inventory. What does it mean in relation to flood forecasts and flood control? What are the storm intensities and frequencies to be expected in the light of the additional meteorologic and hydrologic experience acquired during the past twenty-five years? Are floods foreseeable? Are the results in loss of life and property damage to be charged to the "Act of God", or to the myopia of Man? Reduction in the extent of flood disaster can come through three procedures: (*a*) Information as to maximum flood probability and frequency; (*b*) local forecasts in conjunction with the initial occurrence of storms or snowfall; and (*c*) flood remedial works.

With the foregoing, comes the questions as to what agency—private, local, State, or Federal—is best fitted to handle this large task of study and

[3] *Water Supply Paper 520–G.*

[4] Cons. Engr., Chicago, Ill.

[4a] Received by the Secretary March 23, 1936.

research work? Many State authorities, as well as individuals, have made useful contributions applicable to practical flood relief. The most outstanding, even to-day, are the reports made about twenty years ago by the Miami Conservancy District, a local organization. The salient feature of this work was due to the recognition that flood forecasts, even for local application, depend upon a nation-wide study of hydrological and meteorological data. It is not to be expected, however, that agencies concerned with, and supported by, a single locality or State will continue to undertake a work of national scope. The writer is of the opinion, therefore, that both the cost and supervision of collecting flood data and preparing technical reports for practical flood forecasting and relief can be handled, for the best interest of all concerned, by agencies of the Federal Government. Such work is not routine, and the Government should utilize the paid services of experienced consultants.

This discussion is presented with the idea that action will be taken by the Federal Government, and with the hope that the Society's Committee on Flood Protection Data will continue as a critical, but constructive, adviser to the Government.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

PRINCIPLES TO CONTROL GOVERNMENTAL EXPENDITURES FOR PUBLIC WORKS

FIRST PROGRESS REPORT OF COMMITTEE OF ENGINEERING-ECONOMICS AND FINANCE DIVISION

Discussion

BY MESSRS. EDWARD W. BUSH, FRED LAVIS, AND HORACE H. SEARS

EDWARD W. BUSH,[3] M. AM. SOC. C. E. (by letter).[3a]—It is thought that most engineers will agree that there is a need for the placing in force of the Committee's admirable set of principles when either Federal or State expenditures are made for public works. Some of the discussion at the January 15, 1936, meeting of the Engineering-Economics and Finance Division plainly indicated the need for the application of such principles to State expenditures, and it is hoped that the Committee will favor the Society with its recommendations on what kind of an authority should be established in order to meet the needs of a State; and, furthermore, that it will give the Society more specific information as to what is needed as a Federal authority to control the planning or approval of Federal projects.

In the remarks discussing the principles, the Committee refers to constructions like water supplies, sewerage systems, parks, schools, hospitals, city halls, etc., that are of the kind that, in general, are classed as municipal constructions which, in practically every case, are built under the provisions of State statutes or city charters and ordinances, and it is not clear how its proposed Federal agency could extend its authority to control such constructions. Is it not advisable to divide the subject into what is needed in the way of an organization to control: (*a*) Federal projects; and (*b*) State projects?

Principle (1) states that a program should be "formulated" by the agency, but this would exclude the agency from giving consideration to any program formulated by any other party no matter how worthy such program is of approval. Is it not better to add the words, "or approved", after the word, "formulated"?

NOTE.—The Progress Report of the Committee of the Engineering-Economics and Finance Division on Principles to Control Governmental Expenditures for Public Works, was presented at the Annual Meeting, New York, N. Y., January 15, 1936, and published in February, 1936, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[3] Engr., Fidelity and Surety Dept., Aetna Casualty & Surety Co., Hartford, Conn.

[3a] Received by the Secretary February 26, 1936.

Although every one will agree that the personnel of the agency should be of the same high character and standing as that forming the U. S. Supreme Court it is debatable whether the members of the agency should be appointed for life as are the members of the Supreme Court. The members of the agency will be required to lead a much more active life when investigating the projects before them. Inspection trips may be considered desirable at times, hearings may be held at distant locations, and it is difficult to visualize a board of this kind being able to operate by acting solely on the information it receives by the means of documentary evidence, especially as it would be charged with the duty of formulating programs. A 15-yr term is suggested for the members, each of whom should be, say, from 45 to 55 yr of age when appointed.

At present, most of the Federal departments, bureaus, or authorities controlling expenditures for public works seem to operate more or less independently of one another regarding procedures, or even the contract conditions, when letting contracts. During the past two years old procedures have been canceled and new ones established every few weeks or months so that the contractors and their sureties have been unable to assume that any of the old-time tested economic methods are incorporated in a new contract case without making a close study of the papers to ascertain the actual conditions and obligations that will prevail. Some of the new ideas have been most confusing and grossly uneconomic to the industry, and of a character that produces uncertainties in the minds of the bidders as to what extent they affect the costs of doing the work. It has not been unusual to find, at any one time, many different projects on the market for bids and each having a different set-up regarding matters concerning which there should be little if any difference of opinion among experts in the construction industry. Even in the old days the procedures varied greatly between the different departments and bureaus. If an agency is charged with the duty of formulating or approving the programs it should also be charged with the duty of formulating or approving the general procedures and conditions of all Federal construction operations. Therefore, another principle should be added to the subject herein discussed worded along the line of the following:

> Economic advantages would accrue on Federal contracts if standard forms and procedures in letting contracts were adopted. The agency should be the authority to formulate or approve such measures.

FRED LAVIS,[4] M. AM. SOC. C. E. (by letter).[4a]—During the past year or two, extraordinary expenditures have been proposed and are now being made for the construction of public works by governmental agencies and with governmental funds. These expenditures have been made mostly under the auspices of the Federal Government, but they have also been sponsored, in some degree, by both State and Municipal Governments. Such expenditures, however, have been made to meet an abnormal situation, to relieve a serious unemployment condition caused by a deep depression and setback in the general trend of financial and economic development.

[4] Cons. Engr., New York, N. Y.

[4a] Received by the Secretary February 26, 1936.

Opinions as to the wisdom or lack of wisdom of the distribution of these funds and the merits of the projects for which they have been used, have been very largely colored, by local relief needs and local prejudices, and there have been criticisms that political influence has played a large part in the determination of their distribution and allocation.

With this abnormal situation the present discussion is not concerned. The Committee envisages a much wider field and more general application of the principles it advocates than the temporary condition with which the country is now desperately struggling. The word "desperately" is used advisedly, because the present problem has been so acute, so great, and measures of relief have had to be devised so hurriedly that nothing less than that word describes it adequately.

To repeat, therefore, the report and the recommendations of the Committee are not intended as a standard by which to gauge the merits of projects for temporary relief but, primarily, to set up a measure by which proposals for the expenditure of governmental monies for the construction of public works can be gauged under the orderly, normal conditions which ordinarily prevail. They are to be applied to the every-day normal routine of governmental operations under reasonably usual conditions.

Of course, it goes without the saying, that if such measures become the established criteria in normal times, and are accepted as reasonable criteria because of their wisdom, they undoubtedly will carry over in some degree in judging proposals under abnormal conditions. These latter, however, must always introduce other factors.

The Committee has not been influenced by partisan politics, nor by the slightest animus against the present or any past administration. It offers a suggestion which it believes is in the interest of good government by any political entity.

It will be noted that, although the Principles stated in the report may apply to all public works, they are indicated to apply more definitely to public works to be undertaken by the Federal Government, in somewhat lesser degree by State Governments, and in still lesser degree by Municipal Governments.

In the case of the lesser forms of government, services are developed, which more nearly affect one's personal every-day domestic living, and other criteria enter into consideration, which are of a more intimate personal nature.

It is to be noted that Principle (1) refers to "sound economic considerations." One of the cardinal guiding principles of the Engineering Profession is that, to be a real engineer, one must, fundamentally, be a true economist. The word "economist", to-day seems to be getting into almost as much disrepute as "efficiency expert" did some years ago, not because either term is bad in itself, but because it has been taken up and "tacked on" to so many people who only know a certain pattern of words.

There are economists of the old school who believe in the fundamentals as they know them, and there are economists of the hundred-and-one new schools who feel that nothing but a new set of formulas will fit modern condi-

tions and that these latter (and, consequently, the formulas) are changing with every passing hour.

At the moment the writer is not arguing that either of these schools is right or wrong, although, personally, he still leans toward the old fundamental ideas, with perhaps enough of the "yeast" of the new, to force a few bubbles of "air" coming through the mass to keep it aerated. What he does wish to emphasize, however, is the fact that the application of any theory is largely governed by the experience, caliber, and mentality of the person or persons who apply it and that, of course, is the guiding thought behind Principle (1) which suggests the setting up of an agency comparable in point of experience, dignity, and (ultimate) tradition, with the Supreme Court of the United States.

The laws are there for every one, but the people have come to place great reliance on their final interpretation by a body of men who have wide experience, maturity of thought, dignity, unblemished fearless character, and now, after all these years, a great tradition.

The members of the various legislatures are burdened with a mass of duties. It seems absolutely impossible that they could have either the knowledge, or the time, requisite for an adequate and proper study of the merits of the mass of projects of a public works character, which are brought to their attention for which, in the last analysis, they must be responsible, and for which they appropriate money which the people must pay as taxes. Eventually, they would welcome a tribunal such as that proposed, to which all these projects may be submitted for a truly authoritative study and unbiased opinion. Such a body, of course, would have no authority to take final executive action for or against any project, and would be only advisory as to its usefulness and practicability. If it were composed of experienced, broad-minded, far-seeing, well-informed engineers, it should be of almost inestimable value in lifting some of the present heavy burden from the shoulders of legislators.

There have been suggestions that this advisory body be selected from the permanent personnel of the Bureaus and Departments of the Federal Government. Although this solution might be better than no advisory body at all, it would completely nullify the Committee's proposal that the Board be a judicial advisory body entirely independent of departmental control and able to exercise and establish independent thought and expression without fear of incurring the displeasure of some departmental superior. It is this lack of independent thought and free expression which is to-day building up almost a state of socialism in governmental administration.

If engineers are to maintain and advance the position they have made for themselves as advisers in technical matters, they must also add to this a reputation for ability to judge, accurately, the value of engineering works as parts of the structure of the entire country, or of other countries in which they may be called on to operate.

It is not sufficient to know what particular type of dam will be stable and will properly hold back the water of a reservoir. The engineer must know whether, when the construction of such a dam is proposed, its building is

justified. It may be that it will prevent destructive floods; it may enhance some scenic effect; or it may provide water for domestic use, for irrigation, and for the production of power. If it does any one or more of these things, the engineer should know whether the doing is worth while; whether the cost is justified by the benefits produced, be these social or commercial; and, furthermore, whether or not a similar result may not be obtained more readily by other means.

If public funds are to be used to build highways, the engineer should know for what kind of traffic they are to be constructed, not only what kind of traffic there is to-day, but what there may be during some part at least of the reasonable future life of the road. He should know who is to be benefited by the construction of the road. Will it be all the citizens of the United States, or only some of them? Will it have only local use? Is it to be used only for pleasure cars, or also by heavier vehicles, and what proportion of each? Is it a scenic highway to give pleasure and recreation to all citizens? Is it commercial? Will it enable a farmer or manufacturer better to reach a market? Is it a main artery of transportation, or is it a feeder line? Should it be curved and undulating, fitting the natural topography of the country, or should the hills be cut down, the valleys filled, and the alignment made straight? If so, why, how, and to what extent, will all or any of these conditions affect either the traffic, the cost of operation, or the safety of the vehicles and citizens using this route?

It should be borne in mind, also, that these determinations are seldom subject to solution by mathematical formulas. They are to be resolved exclusively by engineers who not only have a proper background of technical knowledge, but who are also men of vision, wide experience, and good judgment. Public funds, of course, may be expended quite legitimately for such purposes as the creation of parks and the preservation of scenic beauty, and a true interpretation of economics would not bar such expenditures. In consideration of projects of this nature, a competent, unbiased, and independent Board of Engineers would undoubtedly be found useful in the study of many features connected with the acquisition of lands and their future development, the effect of water-sheds, that of forests on rainfall, of floods and droughts, of soil erosion, and the thousand and one factors that affect the physical structure of the land.

In the writer's estimation, these are some of the criteria, very, very briefly indicated, to be applied to the expenditures of public monies. There is really nothing new about them, they have been applied for many years in some degree and with some rigorousness to commercial enterprises where, as one of the older members of the profession expressed it, one had to make "a dollar earn the most interest."

This commercial application is not, however, quite broad enough to be applied to governmental expenditures, some of which quite justifiably may be for pleasure and recreation, or even for military or other public use. The economic principles which must be applied are fundamental (however trite this may sound), but they must be applied intelligently and in the light of ever-changing knowledge.

This will probably never be better expressed in a few words than it has been in the old definition of engineering: "The intelligent development of the forces of Nature, or the sources of power in Nature, to the use and benefit of mankind." The need, to-day, is for some emphasis on the last part of this old definition. Such development must not only be for the use and benefit of mankind, but it must also be a development created in the most intelligent and far-sighted manner with a very clear and unmistakable appreciation of the benefits it is to produce.

In recent years considerable has been heard of relief expenditures for public works as a substitute for the dole. This idea, like many others embodied in catch phrases, perhaps has some element of truth in it. No greater damage, however, could possibly be done to the poor man than to increase his cost of living—a consequence which inevitably follows uneconomic public spending.

Taxpayers, of course, may finally make their voices heard, but then it is usually too late. The money has been spent and they have to pay the bills. The time for the taxpayers to be vigilant is before and not after, and the only way this vigilance can be assured is by some such method as that which is suggested in this report.

The proposal made by the Committee is a long step forward in the right direction; it is very fitting that the Society should have the honor of leading and crystallizing thoughts in a matter of this kind, and perhaps of playing an important part in formulating and possibly setting up a policy of the Federal Government. It is a matter which should be pressed energetically.

Horace H. Sears,[5] M. Am. Soc. C. E. (by letter).[5a]—The following comments are offered as discussion of this report.

Principle (1).—The proposed Federal Public Works program should be based on sound economic considerations and should have a continuity assured by Congress as in the agency appointed for 'fair trade", known as the Federal Trade Commission. The supervision of governmental expenditures by an agent of Congress whose membership has the non-partisan character, and the permanent features of the Federal Trade Commission, might be termed the United States Program Authority, for purposes of this discussion. The Federal Trade Commission is a non-partisan agency. Its members are appointed by the President with the consent of the Senate; and it was authorized to hold public hearings and make investigations for the guidance of the National Industrial Recovery Act.[6]

It should be borne in mind that the Constitution places the judicial powers of the Government in the Supreme Court and such inferior Courts as Congress may establish. In addition to this judicial branch of the Government, the Constitution has granted Congress legislative powers, and the President executive authority. These three departmental powers must be considered in proposing legislation authorizing a new Authority or Com-

[5] Cons. Engr. and Attorney-at-Law, New York, N. Y.

[5a] Received by the Secretary January 15, 1936.

[6] Federal Trade Commission: Title 15; U. S. Code, Sections 41, 46, and 77; also, see. Title 28; Judiciary; Section 334.

mission. Within the past two years (1934 and 1935) the Supreme Court has made several decisions which define the Court's interpretation on the power of Congress to delegate its authority; furthermore, these decisions have limited what may be termed the delegation by the States to the Government of certain specific powers stated in the Constitution.[7] Two decisions rendered by the Supreme Court in 1935 will serve to illustrate the care needed to legislate powers which the Congress has to the proposed U. S. Program Authority: (1) Section 9(*c*) of the National Industrial Recovery Act was held unconstitutional because it delegated in too general terms the powers of Congress over interstate commerce to the President[8]; (2) on December 9, 1935, the Supreme Court held Section 5(*i*) of the Home Owners Loan Act unconstitutional because it conflicted with the rights of Wisconsin to define its own State banking laws.[9] The Supreme Court now has under consideration several suits which contain material pertinent to this proposed U. S. Program Authority; namely, loans by the Public Works Administration to finance municipal revenue utilities[10], and the right of the Security and Exchange Act as amended in 1934 and 1935 to control interstate and intrastate public utilities.

Federal funds are secured from taxes. Once in possession of these revenues, the Government program for spending is uncontrolled, and the Congress has seldom been held in check by the Supreme Court.[11] Existing rights to spend Federal money must be considered in the activities represented by the Reclamation Service[12]; Federal Power Commission[13]; and the Reconstruction Finance Corporation.[14] The foregoing distinction for the use by the Government of funds in hand and its use of proposed expenditures forms a very vital reason for the formation of the proposed U. S. Program Authority. The present status of the unbalanced Federal budget calls for supervision of expenditures for engineering construction by a non-partisan agency of the Federal Congress.

Principle (7).—Costs of engineering projects financed with Federal money should be supervised by an agency of the Government as proposed by this non-partisan U. S. Program Authority. The May, 1935, Report of the Engineering-Economics and Finance Division contained Principle (3) which refers to duplication of costs in municipal revenue-producing plants which compete with existing public utilities. This form of competition is certain to impair outstanding bonds of the utility and the city's revenue bonds as well.[15] State

[7] Kilbourn *vs.* Thompson, 103 U. S. Reports, p. 168.

[8] Panama Refining Co. *vs.* Ryan; U. S. Supreme Court, January 7, 1935, pub. by Commerce Clearing House, "New Deal Decisions—1935."

[9] U. S. Constitutional Amendment X; see, U. S .Supreme Court, December 9, 1935, affirming Wisconsin Supreme Court decision of December 11, 1934, which reversed the County Trial Court, Milwaukee County, September 9–11, 1934. (127 CCH Fed BK Service, Paragraph 18, 419).

[10] Tennessee Valley Authority litigation; U. S. Supreme Court decision pending, January, 1936.

[11] United States *vs.* Realty Co., 163 U. S. Rep., 427.

[12] Title 43; U. S. Code; Sections 391–400.

[13] Title 16; U. S. Code; Section 794.

[14] Title 15; U. S. Code; Sections 605(*a*).

[15] Typical commercial bond (municipal) offerings; see, for example, "The Legal Investment Status of New Issues" (a State legal list) and "Important New Offerings by Dealers" (an attorney opinion) in the *Daily Bond Buyer*, December 21, 1935, pp. 3572 and 3575, respectively.

and Federal Court records for the past two years indicate a large number of injunction suits brought by electric power corporations to protect their franchise to operate under State power grants, and State public service commission rates, whereas the Public Works Administration supplies gifts and loans to permit cities to compete by constructing revenue power plants which are not subject to audit as within the city's constitutional debt limit. As security for loans from the Government, the city issues its revenue bonds which are merely a pledge to pay the loan if and as revenues are collected. The future resale of these revenue bonds to the investing public carries no greater assurance of payment than was given the Government agency (that is the Public Works Administration). On November 21, 1935, the Reconstruction Finance Corporation offered at public sale more than \$5 339 000 of municipal bonds. The individual amounts range from \$3 750 of school district bonds in Missouri, to an issue of more than \$2 500 000, the principal amount of Los Angeles (Calif.), School District bonds. Many of the bonds offered represent public improvements for which an engineer's report should be furnished. Each of the bond offerings of November 21 has a reference to the legal opinion, but as to an engineer's report, no mention is made. This is due to the custom of offering municipal bonds with an opinion as to legality, omitting any reference as to the project's condition from the standpoint of its need for renewal or repairs. A first mortgage on a residence calls for inspection as to its depreciation and need for renewal and repairs after three years from the date of the loan. A greater need exists for an engineer's report on public improvements which were financed from bond issues over a period of 20 yr and now are to be refinanced for an additional period of 10 or 20 yr. Investors in securities which finance engineering costs do not want to buy into a lawsuit. Existing conditions give every assurance that future defaults will occur on bonds which are issued for public improvements without having a suitable engineering plan designed for maintenance and renewal.

In December, 1935, the Securities and Exchange Commission ruled that registered security dealers must print on each prospectus the statement that the Commission does not guarantee directly or indirectly the securities offered. The application requirements made by the Commission have been satisfied in part by having a report from an engineer or from a public accountant as the case might be, as evidence that the security dealer had made proper investigation as to the construction and financing of the project for which the securities were issued. It should be noted that municipal securities are exempt from this registration and that neither the registration of securities nor the resale by the Government of securities owned by the Public Works Administration will prevent a purchaser from suing for damages when the dealer knowingly sells securities by the use of fraudulent statements.[16]

[16] Operations under the Federal Securities Act: Financial Management Series FM 46; copyright 1935, by American Management Assoc., (330 West 42d Street), New York, N. Y. The 1934 Securities and Exchange Act, amending the 1933 Act, transferred this Commission from its original agency with the Federal Trade Commission.

In conclusion it should be stated that Federal legislation may be had, as a matter of powers of Congress to designate a United States Program Authority. The existing use of Federal money—(*a*) in financing city, county, and State engineering projects; (*b*) in assisting to refund matured or defaulted municipal securities; and (*c*) in future programs for public improvements—calls for a National organization that will educate the seller and buyer of municipal securities to demand an engineer's report to the same extent that in the past has been the case in offering an attorney's opinion. Legal lists authorized by States, of selected securities for trust investments, should include an engineer's report when engineering projects are financed from the sale of municipal securities.